Cartography and Geographic Information Science, 2013
http://dx.doi.org/10.1080/15230406.2013.795002

Blending world map projections with *Flex Projector*

Bernhard Jenny[a]* and Tom Patterson[b]

[a]College of Earth, Ocean, and Atmospheric Sciences, Oregon State University, Corvallis, OR 97331-5503, USA; [b]US National Park Service, Harpers Ferry Center, Harpers Ferry, West Virginia 25425-0050, USA

(*Received 12 October 2012; accepted 15 February 2013*)

The idea of designing a new map projection via combination of two projections is well established. Some of the most popular world map projections in use today were devised in this manner. One construction method is to combine two source projections along a common parallel; a second method calculates the arithmetic means of two projections. These two methods for creating new world map projections are included in the latest version of *Flex Projector*. *Flex Projector*, a freeware mapping application, offers a graphical approach for customizing existing projections and creating new projections. The *Mixer* is a new feature in the latest version that allows the user to blend two existing projections to create a new hybrid projection. In addition to the two established combination methods, the software includes a new method for blending projections specific to its visual design approach. With this new method, a unique trait of one projection is transferable to a second projection. *Flex Projector* allows for the blending of four different projection traits separately or in combination: (1) the horizontal length of parallels, (2) the vertical distance of parallels from the equator, (3) the distribution of meridians, and (4) the bending of parallels. This article briefly describes the main characteristics of *Flex Projector* and then documents the new approaches to projection blending. The integration of the three methods into *Flex Projector* makes creating new projections simple and easy to control and allows the user to evaluate distortion characteristics of new projections. As an applied example, the article also introduces the new Pacific projection that is a blend of the Ginzburg VIII and Mollweide projections.

Keywords: projection design; projection blending; *Flex Projector*; Pacific projection

Introduction

Flex Projector (www.flexprojector.com) is a free, open-source, cross-platform application with a graphical user interface for designing world map projections. This article discusses the *Mixer*, a feature in *Flex Projector* for selectively combining two projections. The *Mixer* complements the other design tools found in *Flex Projector* and further simplifies the creation of new world map projections.

When developing *Flex Projector*, the goal was to give users without expert knowledge in mathematics an accessible tool for designing world map projections. The application creates pseudocylindrical and cylindrical projections, as well as projections with curved parallels. It allows users to shape the graticule and provides visual and numerical feedback for assessing distortion properties. The design of the graphical user interface was done from the end user's perspective – ease-of-use and encouraging experimentation were priorities (Jenny and Patterson 2007; Jenny, Patterson, and Hurni 2008). The intended users of *Flex Projector* are practicing mapmakers and cartography students. Details for the mathematics and algorithms that convert user settings to formulae for projecting digital data are covered in Jenny, Patterson, and Hurni (2010).

The inspiration for developing *Flex Projector* was Arthur Robinson's graphical approach to projection design. In 1961, working on a commission for the Rand McNally publishing house, Robinson created his eponymous world map projection, originally dubbed the orthophanic, meaning correct-looking (Robinson 1974). Robinson proceeded through an iterative process to create his pseudocylindrical projection, graphically evaluating the appearance and relative relationships of landmasses. He first estimated the values for parallel lengths and spacing, then the projection was drawn and the continents plotted. When he found early drafts less than satisfactory, compensating adjustments to the graticule were made and the continents replotted. This iterative process, a sort of graphic successive approximation, was repeated until it became obvious that further adjustments would produce no improvement, at least to the eyes of the author (Robinson 1974, 151–152). Others agreed with Robinson. His projection has since become widely popular for making world maps, used by National Geographic Society (Garver 1988) and other respected cartographic establishments.

The appeal of the Robinson projection is due in large part to the pleasant appearance of the graticule and the

*Corresponding author. Email: jennyb@geo.oregonstate.edu

major landmasses. It presents the world in a handsome, partially oval container; the continents within it look correct in size and shape to most readers. The success of the Robinson projection is due largely to the fact that it is a compromise projection, that is, it preserves neither angles nor areas. Because designing projections always involves compromises, a projection by necessity must distort geographic shapes, often grossly. Conformality (the preservation of angles) is a property ill-suited to general world maps (Canters 2002). In contrast, most cartographers value the equal-area property, as the comparison of areal extents is made easier. Additionally, some cartographic methods require an equal-area base, for example, choropleth maps (showing values usually normalized by area by differently shaded areas) or dot maps (where the relative density of dots changes with areal distortion). However, when strict adherence to the equal-area property is not required, a compromise projection often shows the shapes of continents with a more pleasant appearance than equal-area projections (Canters 2002). *Flex Projector* and its *Mixer* feature allow cartographers to design compromise projections that balance the competing priorities of equal-area fidelity and pleasing appearance.

This article extends this graphical approach and introduces graphical tools for blending existing projections to create new world projections. Three different methods for blending projections are included that offer complementary approaches to the design of world map projections and are often faster and easier to control than the original method. *Flex Projector's* other functionality and its graphical user interface are described in more details by Jenny, Patterson, and Hurni (2008), whereas Jenny, Patterson, and Hurni (2010) documents the mathematical background and the visualization of distortion characteristics.

The discussion is structured as follows: First, we examine existing methods for combining projections applied in the past to create a variety of projections. We identify three groups: combining along lines of latitude, arithmetic means, and interpolating with varying weights. The second section discusses three methods implemented in *Flex Projector*, including a new approach allowing the user to combine selected traits of two projections to create a new projection. The last section then discusses the design and characteristics of the Pacific projection, before we conclude with a few final remarks.

Flex projector for customizing projections

Upon opening *Flex Projector*, the user sees a graphical user interface comprised of three components (Figure 1). The panel in the upper left is a world map in the Robinson projection, the default. To the right of the map is a panel with sliders that control the shape of the projection and tempting the user to experiment. Moving any of the sliders results in an immediate change to the Robinson projection, which then ceases to be a Robinson projection and starts on its way to becoming an entirely new projection. Four groups of sliders exist, for adjusting the length of parallels, their vertical distribution, their bending, and the distribution of meridians. Below the map is a table with distortion

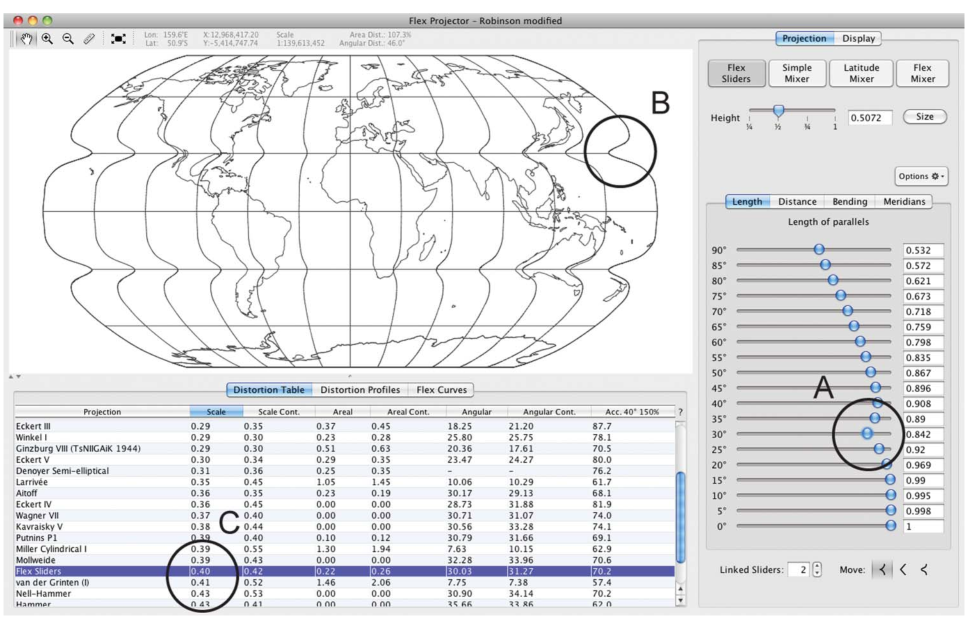

Figure 1. Screenshot of *Flex Projector*: moving sliders change the length of parallels based on increments of five degrees of latitude (A), which in turn changes the projection shape (B) and the distortion ranking (C).

indices, which reports in real-time the amount of distortion contained in the modified projection, including comparisons to common world map projections. These distortion indices, as well as specialized distortion visualization techniques, have been documented before (Jenny, Patterson, and Hurni 2010). The tools in *Flex Projector* provide a means to design projections in the same manner as Robinson did nearly 50 years ago – more accurately, quickly, and with much less tedium.

Combining projections

The idea of designing a new map projection by combining two existing projections is well established. Some of the most popular world map projections in use today were devised in this manner. In general, the goal is to merge the desired characteristics of two projections, while eliminating disadvantages. For example, the pointed poles of the sinusoidal projection add considerable angular distortion to polar areas, while projections with a polar line, such as the Robinson projection, introduce less shape distortion at poles. To date, cartographers have developed various techniques for combining world map projections, which can be grouped in three categories, as described below.

It remains to be mentioned that a variety of alternative methods for modifying a single projection exist. Canters (2002, 115ff) distinguishes between polynomial transformations for projections and the modification of projection parameters, including Wagner's powerful *Unbeziffern* (or re-numbering) method (Wagner 1949). These methods modify a single source projection to create a new projection and do not combine two source projections.

Combining projections along lines of latitude

Projections in the first group are hybrid projections made by fusing together parts of other projections. For world map projections, this typically involves joining two pseudocylindrical projections along a common parallel. For example, Goode (1925) combined the Sanson sinusoidal and the Mollweide projection at 40° 44′ 12″; north and south latitude, which is the latitude of equal scale. The resulting Goode homolosine projection is most common in its interrupted form. Others proposing non-continuous combined projections include, for example, Érdi-Krausz (1968), Hatano (1972), and McBryde (1978) (also see Canters 2002, 154 and Snyder 1993, 217–220, for overviews). A trait of most non-continuous combined projections is a discontinuity in the first derivatives at the latitude where the two projections join. This typically appears as a sharp crease where the meridians meet. This discontinuity can be visually disturbing, especially when it is concave, as is the case for Goode's homolosine projection. For some source projections, mathematical methods exist for eliminating the discontinuities in the destination projection. For example, Gede (2011) eliminates the visual join of the Érdi-Krausz projection.

Arithmetic means of two projections

Calculating the arithmetic means of two different projections is the technique for devising a large number of projections. The two "starter" projections are often a cylindrical projection, such as the plate carrée and a pseudocylindrical projection with meridians converging at pole points. Examples include projections devised by Eckert (1906), Putniņš (1934), and Winkel (1921). For example, the Winkel Tripel projection is the arithmetic mean of an equirectangular projection and the Aitoff projection (Winkel 1921; Snyder 1993, 231–232); Eckert V is an average of the plate carrée and the sinusoidal (Eckert 1906). Foucaut (1862), Hammer (1900), Nell (1929), and Tobler (1973) have averaged the cylindrical equal area and the sinusoidal (after Snyder 1977). Some of these projections are equal area, which is possible if either the x or y coordinate is obtained by averaging the two source projections and the other coordinate is mathematically derived from the equal-area condition (Tobler 1973). This is also the technique used by Boggs (1929) for his combination of the sinusoidal and the Mollweide.

Interpolating projections with varying weight

Interpolating projections using variable weighting is an extension of the previous technique. In examples discussed by Anderson and Tobler (s.d.), the imposed weighting varies with the latitude, decreasing from one at the equator to zero at the poles. Tobler (1973) has applied this technique to create various equal-area projections.

Combining projections with the Flex Projector mixer

Flex Projector offers three methods to combine projections. The first method joins two projections along a selected latitude, the second method computes an arithmetic means of two projections, and the third method is a new approach to combine selected characteristics of two projections. As described in the introduction of this article, *Flex Projector* aims at providing a graphical approach to the design of map projections. For all three methods, the user loads two map projections from pop-up lists on the right side of the main window (Figure 2). Moving the sliders at the top right interactively combines the loaded projections. Moving a slider to the left or right proportionally controls the influence of each projection. As the user experiments, changes appear instantaneously on the large composite world map (Figures 2–4).

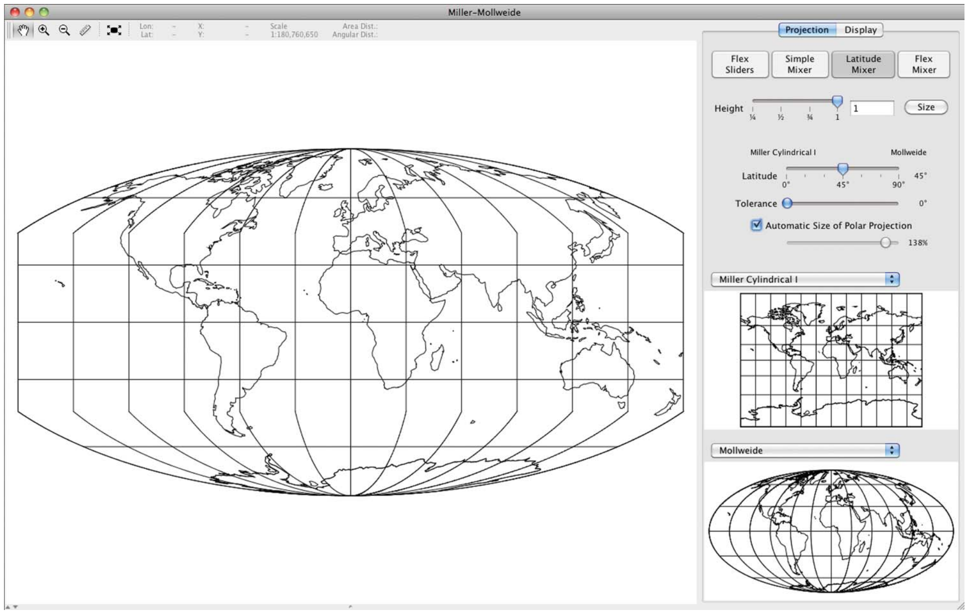

Figure 2. *Latitude Mixer* panel in *Flex Projector* combining the Miller Cylindrical I (center right) and Mollweide (bottom right) projections at 45° latitude. (The resulting projection is a dramatic example of the technique and not intended for actual mapping.)

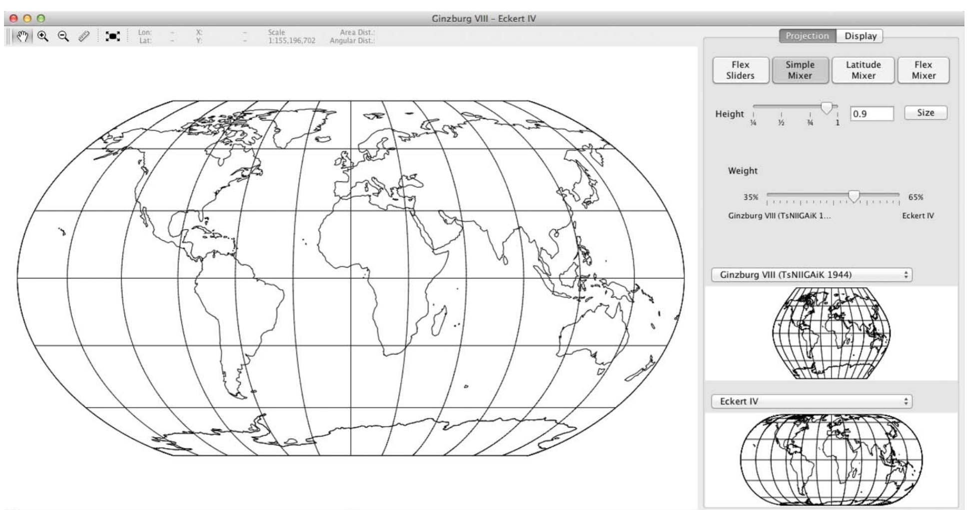

Figure 3. A projection created by computing the weighted means of the Ginzburg VIII (35%) and the Eckert IV (65%) projections and scaling vertical coordinates by 0.9.

Combining projections along lines of latitude

With the *Latitude Mixer*, the user can choose the latitude along which the two projections are combined. The shared parallel generally does not have the same length with both projections, which requires one of the projections to be scaled. We chose to scale the projection showing higher latitudes. The scale factor is normally computed automatically or can be adjusted manually (although this option is probably only useful for didactical purposes). A second slider defines a latitude band for linearly interpolating around the shared parallel, which can smooth the crease along the parallel where the two projections join. A third slider adjusts the height-to-width ratio of the combined projection (Figure 2).

Arithmetic means of two projections

The *Simple Mixer* computes the means of two source projections. For example, the "DNA" of the new projection depicted in Figure 3 is 35% Ginzburg VIII and 65% Eckert IV. The Ginzburg VIII projection was chosen because of its appealing depiction of landmasses at moderate latitudes. The equal-area Eckert IV was chosen to compensate the overly large polar areas of the Ginzburg

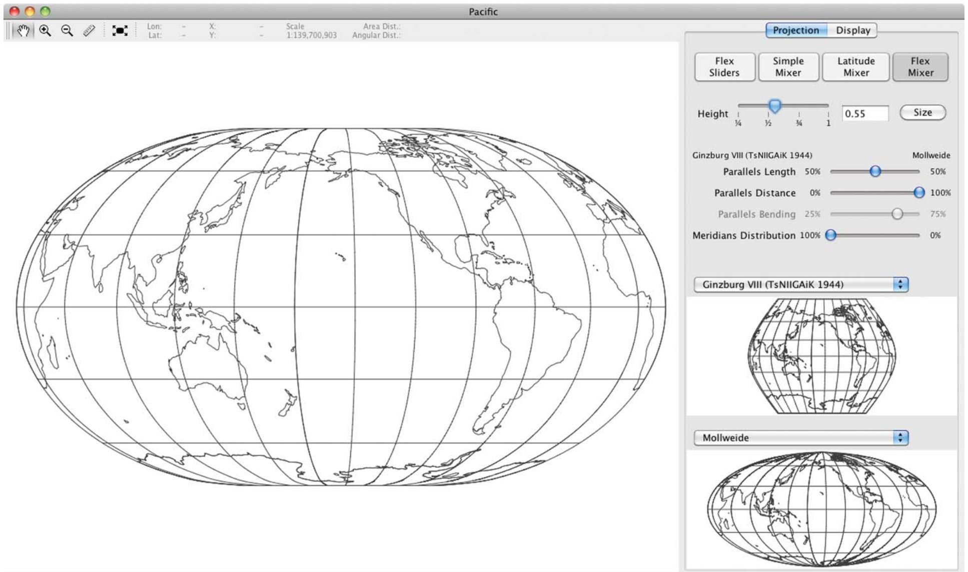

Figure 4. *Flex Mixer* panel in *Flex Projector*: The Pacific projection (left) is a blend of the Ginzburg VIII (center right) and the Mollweide (bottom right) projections. Sliders at the top right control the blending of the three active parameters (with blue buttons) that define the hybrid Pacific projection.

VIII. Additionally, scaling the height-to-width ratio to 0.9 depicts major landmasses with more graphically pleasing proportions (Figure 3).

The authors also experimented with options for graphically adjusting blending weights with latitude, the third idea for combining projections discussed in the previous section. Interactive spline curves – similar to the Curve Adjustments panel in Adobe Photoshop – were added to a prototype version of *Flex Projector*. The curves allowed users to adjust the weight with latitude by adding spline segments to the curve and by adjusting the position of the knots between the spline segments. Because the functionality of this tool is relatively difficult to grasp for novice users and because the effect is difficult to control, the current version of *Flex Projector* does not include this option.

Selective combinations

The *Flex Mixer* selectively transfers a unique trait from one projection to another, such as replacing the straight parallels of the Eckert IV projection with the arced parallels of the Winkel Tripel projection. No other characteristics of the Eckert IV would change.

Four different projection properties can be combined: (1) the horizontal length of parallels, (2) the vertical distance of parallels from the equator, (3) the distribution of meridians, and (4) the bending of parallels. The user can adjust weights using four sliders (Figure 4, top right). For example, when setting the weight for the horizontal length of parallels to 30%, the parallel lengths of the mixed projection are a combination of 30% of the parallel length of the first source projection and 70% of the second projection. The same principle applies to the other three properties. If neither of the two source projections has bent parallels or irregularly distributed meridians, mixing these attributes would not change the final projection and the corresponding sliders are accordingly deactivated.

The simple graphical interface hides an algorithm from the user that proceeds in three steps. In the first step, *Flex Projector* "deconstructs" the two selected source projections by converting them into tabular form. A projection is commonly defined by a pair of transformation formulae with the form $X = f(\varphi, \lambda)$ and $Y = g(\varphi, \lambda)$ that convert longitude λ and latitude φ into projected Cartesian X/Y coordinates. The first step creates four tables of numerical values for each projection using the corresponding pair of transformation formulae. Three of the resulting tables contain values for every 5° of increasing latitude (the length, vertical distribution, and bending of parallels) and one table contains values for every 15° of increasing latitude (the horizontal distribution of meridians). Both source projections are converted to these tabular forms, resulting in 2 × 4 tables.

In a second step, the four pairs of tables are blended using the four user-defined weights. The four pairs of tables are merged to four blended tables by computing a weighted average of each pair of corresponding tabular values.

The final third step converts geographical longitude/latitude coordinates to Cartesian X/Y coordinates. The conversion interpolates spline curves through the values stored in the four tables and then applies an extended version of the method presented by Robinson (1974) for projecting geographic coordinates to Cartesian coordinates (see Jenny, Patterson, and Hurni (2010) for details).

After mixing the four different projection traits with this method, the user may then fine-tune the new projection by adjusting individual values of one of the four tables with *Flex Projector's* graphical user interface, as shown in Figure 1.

It must be mentioned that the described technique does not work perfectly for all projections. The reason is that the first step in the algorithm (the transformation from formulae to tabular values) sometimes does not accurately replicate the original projection. While cylindrical and pseudocylindrical projections with regularly distributed meridians are matched perfectly, only approximate transformations are possible for projections with arcing parallels or projections with irregularly spaced meridians.

The Pacific projection

As a practical example on how to use the *Mixer*, we created the hybrid *Pacific* projection by combining the Ginzburg VIII and Mollweide projections (Figure 4). The design intent was a world map with a rather conventional appearance centered on 160° west longitude to focus on the Pacific Ocean and with relatively little areal distortion. We chose the Mollweide projection because it is oval in shape, which complements the roundness of the Pacific basin, and equal-area. The Ginzburg VIII contributes unevenly distributed meridians that are widely spaced at the projection center and compressed at the map margins, a useful feature for emphasizing the Pacific in relation to other parts of the world. Creating the Pacific projection involved adjustments to three parameters controlled by the sliders at the top right of the *Mixer* panel (Figure 4). Because neither the Ginzburg VIII nor Mollweide projection has parallels that bend, this parameter is disabled in the graphical user interface. Adjustments to the sliders included the following:

Length of Parallels. Setting the slider in the middle at 50% gives equal weight to the Ginzburg VIII and Mollweide projections for this parameter. This combination gives the Pacific projection highly rounded pole lines that merge smoothly into the lateral meridians.

Distance of Parallels. Setting the slider at 100% by dragging it all the way to the right toward the Mollweide projection weighted this parameter entirely from that projection. The Pacific projection, as a result, has a Mollweide-like vertical distribution of parallels.

Distribution of Meridians. Setting the slider at 0% by dragging it all the way left weights this parameter entirely toward the Ginzburg VIII projection, thus increasing the area of the Pacific Ocean because the meridians near the center point are more widely spaced than those at the map margins.

Once work in the *Mixer* is finished, the user can further enhance the combined projection using the other tools in *Flex Projector*. In the case of the Pacific projection, we increased the height-to-width proportions from 0.507 to 0.55 to make the map taller and accentuate the round shape of the Pacific Ocean (Figure 5). Additionally, we scaled the entire map by a factor of 0.8 to minimize area distortion for the Pacific region. By saving the final Pacific projection as a small text file, we could use it again in *Flex Projector* for producing publishable-quality maps with imported shapefiles and raster geodata. However, operational use of the Pacific projection and all custom

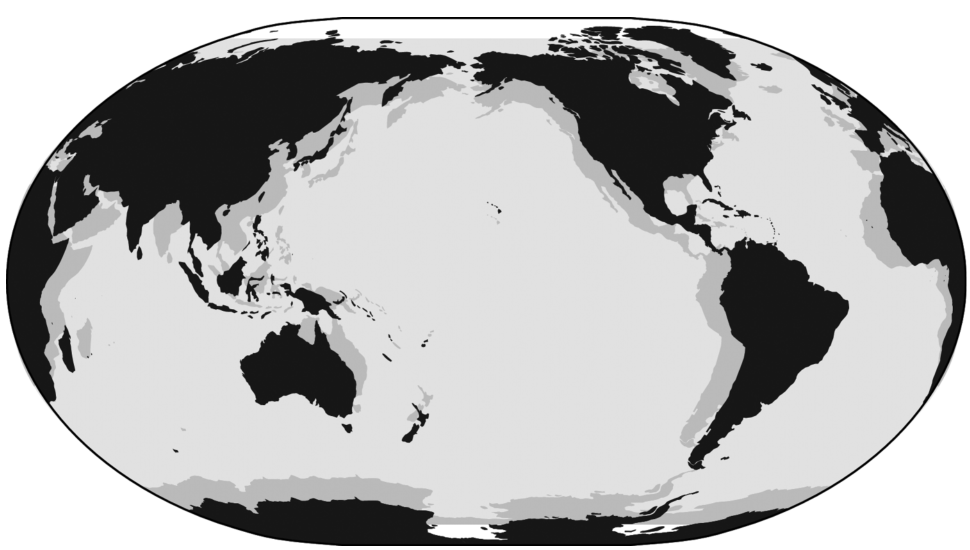

Figure 5. The Pacific projection (black) overlaid on the Robinson projection (gray). The Robinson is scaled to the same width. The taller Pacific projection devotes relatively more area to the Pacific Ocean than does the Robinson.

Projection	Scale	Areal	Angular
Eckert IV	0.36	0.00	28.73
Sinusoidal	0.51	0.00	39.01
Hammer	0.43	0.00	35.66
Kavraisky V	0.38	0.00	30.56
Mollweide	0.39	0.00	32.28
Goode Homolosine	0.46	0.00	36.39
Putnins P4'	0.39	0.00	31.54
Boggs Eumorphic	0.44	0.00	35.20
Wagner VII	0.37	0.00	30.71
McBryde–Thomas Flat-Pole Sine (No. 2)	0.32	0.07	26.38
Putnins P1	0.39	0.10	30.79
Wagner II	0.32	0.12	26.88
Winkel II	0.27	0.17	24.20
Winkel Tripel	0.26	0.18	23.28
Flex Mixer: Ginzburg VIII (TsNIIGAiK 1944) + Mollweide	0.37	0.18	30.68
Robinson	0.27	0.19	21.27
Natural Earth	0.25	0.19	20.54
Winkel I	0.29	0.23	25.80
Aitoff	0.36	0.23	30.17

Figure 6. *Flex Projector* distortion table with the weighted mean errors for scale and area distortion and mean angular deformation indices (Canters 2002). The Pacific projection is the highlighted row.

projections created with *Flex Projector* is not possible outside of the *Flex Projector* environment. Doing this would require programming of custom code or, for some cases, development of the approximating mathematical formulas (Šavrič et al. 2011).

Projections created with the *Mixer* exhibit distortion values similar to the projections from which they derive. For example, because the Pacific projection is part Mollweide projection, which is equal-area, it ranks favorably for areal distortion. It is not free of areal distortion, however, because of the influence of the non-equal-area Ginzburg VIII. The *Flex Projector* distortion tables (Figure 6) show how the Pacific projection ranks against common world map projections for the other distortion categories – it is unexceptional. When comparing the distortion values in Figure 6, lower values are better for all categories. The tint in Figure 7 shows the extent of acceptable area on the Pacific projection. The acceptable area in this example is the area with an angular distortion of less than 30° and areal distortion between 90% and 111%. The isoline maps in Figure 8 show patterns of maximum angular distortion and areal distortion. For both metrics, the amount of distortion is minimized over the one-third of Earth occupied by the Pacific Ocean, including relatively high latitudes. The area distortion isolines in Figure 8 show the compression and expansion of area, with the thick isoline indicating a line without area distortion. The center of the map at 160°W and 0°N has an area distortion of 97.2%, which means that the central part of the map is compressed by less than 3%. The entire Pacific Ocean, up to the latitude of approximately 50° N and S, is displayed with less than 5% area distortion. For a full explanation of the distortion tables and indices, as well as the distortion isolines, refer to Jenny, Patterson, and Hurni (2010).

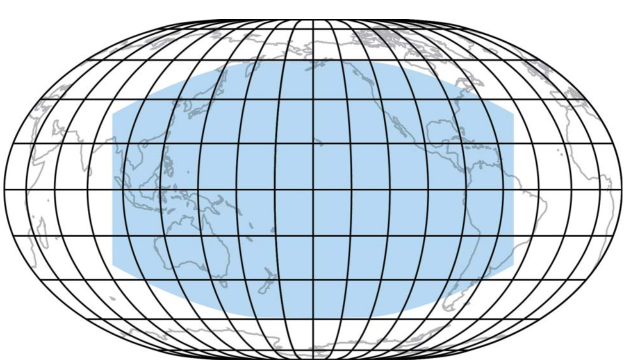

Figure 7. The tint indicates the area of acceptable angular and areal distortion for the Pacific projection. Most of the Pacific with the exception of waters adjacent to Antarctica show angular distortion of less than 30° and areal distortion between 90% and 111%.

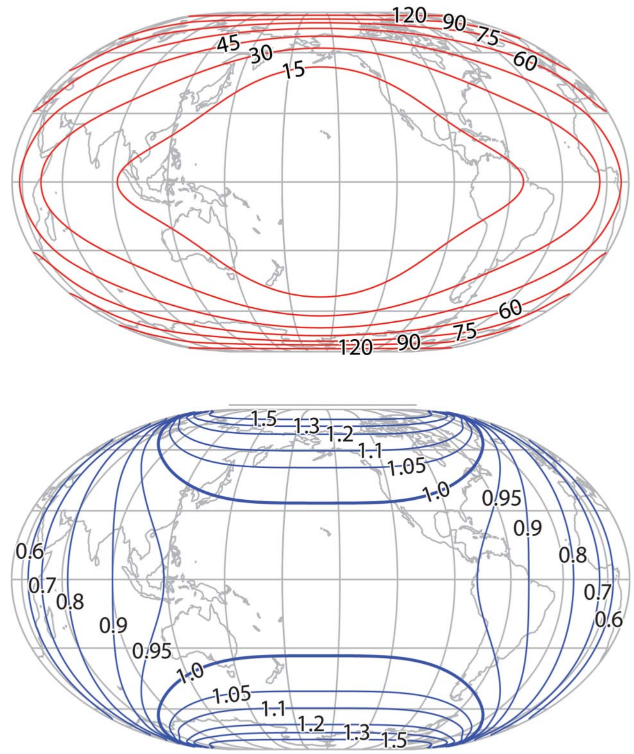

Figure 8. Isolines of maximum angular distortion (top) and areal distortion (bottom) for the Pacific projection.

Conclusion

Flex Projector simplifies the design of new hybrid projections. It can selectively blend individual characteristics of existing projections. If necessary, the blended projection can serve as a good starting point for additional fine-tuning using the graphical user interface for adjusting the curves and other options (as shown in Figure 1). It is also possible to load a projection created in the *Mixer* back into the *Mixer* to combine it again with other projections.

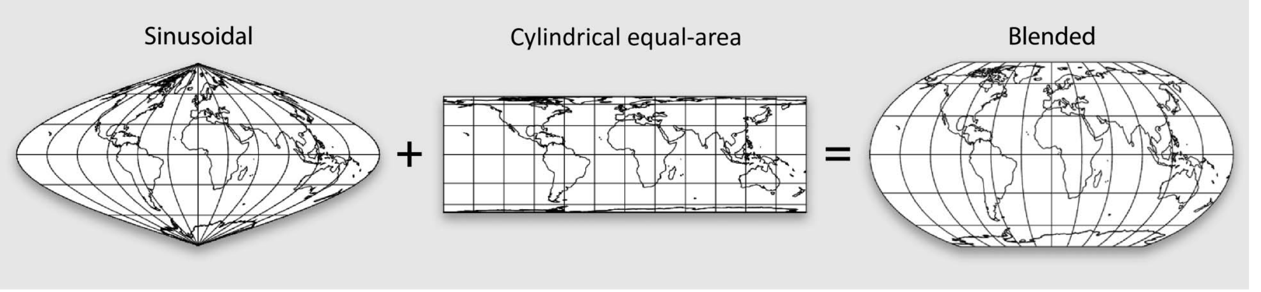

Figure 9. Combining two equal-area projections (left and middle) produces a blended hybrid (right) that is nearly equal-area.

Repeating this process can yield an almost infinite variety of new projections.

In the *Mixer*, it is easy to design new projections that not only are visually pleasing, but also have excellent distortion characteristics. For example, the two equal-area projections blended in Figure 9 yield a new pseudocylindrical projection with an overall shape similar to the Robinson projection and with less areal distortion (0.05 vs. 0.19, 0.0 is equal-area), albeit at the expense of additional deformation for continental shapes. According to Anderson and Tobler (s.d.), "blended map projections are splendid projections." We think that users of the *Flex Projector Mixer* will come to the same conclusion.

Acknowledgments

The authors thank the anonymous reviewers for their valuable comments.

References

Anderson, P. B., and W. R. Tobler. (s.d.). "Blended Map Projections are Splendid Projections." Accessed August 3, 2011. http://www.geog.ucsb.edu/~tobler/publications/pdf_docs/inprog/BlendProj.pdf.

Boggs, S. 1929. "A New Equal-Area Projection for World Maps." *Geographical Journal* 73 (3): 241–245.

Canters, F. 2002. *Small-Scale Map Projection Design*. London: Taylor & Francis.

Eckert, M. 1906. "Neue Entwürfe für Weltkarten." *Petermanns Mitteilungen* 52 (5): 97–109.

Érdi-Krausz, G. 1968. *Combined Equal-Area Projection for World Maps, Hungarian Cartographical Studies*. 44–49. Budapest: Földmérési Intézet.

Foucaut, H. C. 1862. *Notice sur la Construction de Nouvelles mappemondes et de Nouveaux Atlas de Géographie*. France: Arras.

Garver, J. B. 1988. "New Perspective on the World." *National Geographic* 174: 910–913.

Gede, M. 2011. "Optimising the Distortions of Sinusoidal-Elliptical Composite Projections." In *Advances in Cartography and GIScience*. Volume 2: Selection from ICC 2011, Paris, Lecture Notes in Geoinformation and Cartography 6, doi;10.1007/978-3-642-19214-2_14, edited by A. Ruas, 209–225. Berlin: Springer-Verlag.

Goode, J. P. 1925. "The Homolosine Projection: A New Device for Portraying the Earth's Surface Entire." *Annals of the Association of American Geographers* 15 (3): 119–125.

Hammer, E. 1900. "Unechtcylindrische and Unechtkonische flächentreue Abbildungen." *Petermanns Geographische Mitteilungen* 46: 42–46.

Hatano, M. 1972. "Consideration on the Projection Suitable for Asia-Pacific Type World Map and the Construction of Elliptical Projection Diagram." *Geographical Review of Japan* 45 (9): 637–647.

Jenny, B., and T. Patterson. 2007. "Flex Projector." Accessed August 3, 2011. http://www.flexprojector.com.

Jenny, B., T. Patterson, and L. Hurni. 2008. "Flex Projector – Interactive Software for Designing World Map Projections." *Cartographic Perspectives* 59: 12–27.

Jenny, B., T., Patterson, and L. Hurni. 2010. "Graphical Design of World Map Projections." *International Journal of Geographic Information Science* 24 (11): 1687–1702.

McBryde, F. W. 1978. "A New Series of Composite Equal-Area World Maps Projections." International Cartographic Association, 9th International Conference on Cartography, College Park, Maryland, Abstracts, 76–77.

Nell, A. M. 1929. "Äquivalente Kartenprojektionen." *Petermanns Geographische Mitteilungen* 36: 93–98.

Putniņš, R. V. 1934. "Jaunas projekci jas pasaules kartēm." [Latvian with extensive French résumé.] *GeografiskiRaksti, Folia Geographica 3 and 4*: 180–209.

Robinson, A. 1974. "A New Map Projection: Its Development and Characteristics." In *International Yearbook of Cartography*, edited by G.M. Kirschbaum, and K.-H. Meine, 145–155. Bonn-Bad Godesberg: Kirschbaum.

Šavrič, B., B. Jenny, T. Patterson, D. Petrovič, and L. Hurni. 2011. "A Polynomial Equation for the Natural Earth projection." *Cartography and Geographic Information Science* 38 (4): 363–372.

Snyder, J. P. 1977. "A Comparison of Pseudocylindrical Map Projections." *The American Cartographer* 4 (1): 59–81.

Snyder, J. P. 1993. *Flattening the Earth: Two Thousand Years of Map Projections*. Chicago, IL: University of Chicago Press.

Tobler, W. R. 1973. "The Hyperelliptical and Other New Pseudo Cylindrical Equal Area Map Projections." *Journal of Geophysical Research* 78 (11): 1753–1759.

Wagner, K. 1949. *Kartographische Netzentwürfe*. Leipzig: BibliographischesInstitut.

Winkel, O. 1921. "Neue Gradnetzkombinationen." *Petermanns Mitteilungen* 67: 248–252.