



## Cultural Heritage

## Studying cartographic heritage: Analysis and visualization of geometric distortions

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## ABSTRACT

Old maps are increasingly used as a source for historical research. This is a consequence of the increased availability of old maps in digital form, of the emergence of user-friendly Geographical Information Systems, and of a heightened awareness of the unique information stored in old maps. As with every source for historical studies, when old maps are georeferenced and information is extracted for historical research, the accuracy and reliability of the geometric and semantic information must be assessed. In this paper, a method based on a series of geometric transformations is presented, which transforms control points of a modern reference map to the coordinate system of an old map. Based on these transformed points, the planimetric and geodetic accuracy of the old map can be computationally analyzed and various visualizations of space deformation can be generated. The results are graphical representations of map distortion, such as distortion grids or displacement vectors, as well as statistical and geodetic measures describing the map geometry (e.g., map scale, rotation angle, and map projection). The visualizations help to assess the geometric accuracy of historical geographical information before using the data for geo-historical studies. The visualizations can also provide valuable information to the map historian about the history of a particular map and its creation.

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## 1. Introduction

Maps and other cartographic artifacts, such as panoramic drawings, globes, or relief models, are an important part of our cultural heritage. They are more than mere physical artifacts; they are also a potential source of information for historical studies. Maps mirror the historical state of geographical knowledge, contemporary ideology, and geopolitical interests at a certain point in time. Often, they are also very esthetic. Scholars in the arts, sciences, and humanities study old maps in the field of the history of cartography in strict sense, for example, to trace the development of mapping techniques, the use of maps, cartographers, or individual maps. Historians and geographers also increasingly use maps as sources for historical studies, where geographical information is extracted from old maps and, for example, used to trace the evolution of a landscape or a traffic network (see, for example, [1–3] for case studies).

The fragile nature of maps requires special care for their storage, preservation, dissemination, and use, which motivates an increasing number of map libraries to digitize their cartographic heritage. A digital map cannot physically decay; it is more

easily accessible by digital download, and its content can be analyzed and interpreted with digital tools. A simple example application would be to locate a settlement that has disappeared over time by overlaying a historical map and a modern reference map. Intangible information, e.g., past toponymy, can also be gathered; in this example it could be an ancient name of the settlement. For more complex geo-historical studies, Geographical Information Systems (GIS) for capturing, storing, analyzing, and representing geographical data are essential tools [4]. Before analysis with a GIS can begin, georectifying the old map is an indispensable step. The map is stretched and rotated such that it aligns as well as possible with a modern reference map or a reference coordinate system. Various types of geometric transformations are offered by GIS software (e.g., global affine transformations, or warping rubber sheet interpolations). Once the map is georectified, information can be extracted from the map for quantitative or qualitative studies to assess, for example, historical land use and agriculture [5,6], demography [7], or long-term change of forest [8].

For a map to be a geometrically reliable source of information, it must be an accurate representation of a real locality. Indeed, not every map is drawn with the intention to represent the geometric situation with the highest accuracy possible, as for some cartographic applications a precise geometry is not required, or space is deformed intentionally to graphically emphasize selected areas.

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Hence, different users and uses call for different accuracy [9]. Some maps even show non-existing landscapes, which may contain misleading information for propagandistic or military purposes, or show projected features that were never realized. In these cases a geometric analysis for historical studies is often pointless, and we therefore disregard such maps in the following considerations.

Before using the information contained in old maps for historical studies, however, its quality must be assessed. According to Blakemore and Harley [9], three different aspects should be assessed. The first aspect is the topographic accuracy that denotes the quantity and quality of information about landscape objects. Every map contains only a selection of geographic features that are selected and symbolized by the cartographer according to the goal of the map. Hence, the question is, whether the map depicts all features of a certain class, and how accurately the cartographer classified the features thematically [10]. The second aspect is the chronometric accuracy, i.e. both the dating of the map as a physical artifact (by watermark analyses or other techniques), and the dating of the information contained in the map. Dating the age of map information is often difficult, as the production or the revision of a map commonly takes several years. The third aspect is the planimetric completeness (or geometric accuracy). This idea was introduced by Laxton [10], who differentiates between the geodetic accuracy and the planimetric accuracy. The geodetic accuracy describes the accuracy of the positioning of the map in a global coordinate system. That is, the accuracy of the mathematical foundation of a map is studied, such as the reference meridian, the map projection, or the ellipsoid modeling the shape of the Earth. The planimetric accuracy – the extent to which distances and bearings between identifiable objects coincide with their true values – is determined by comparing the positions, distances, areas, and angles of features on the map and in reality, allowing for assessment of geometric features regarding their suitability for a historical study.

In the following article, we take a closer look at the geodetic and planimetric accuracy. Apart from supporting scientists in assessing the reliability and accuracy of data extracted from maps for geo-historical studies, planimetric analysis can also help map historians evaluate theories on technical aspects of map creation. The map historian may, for example, verify assumptions on the surveying methods and source maps used to compile the map, or examine the underlying projection and geodetic references. Both types of studies – geo-historical or map historical – require statistical measures and graphical visualizations. Graphic representations of the planimetric accuracy aid the discovery and understanding of new facts about an old map, and are an excellent means of illustrating these findings.

An accuracy visualization can reflect geometric imprecision induced by two sources. First, imprecision may be introduced during the different stages of map production (e.g., surveying and data compilation, map drawing, copying, and reproduction); second, paper, vellum, or other media, which store maps are not inert materials, i.e. shrinking and stretching over time distort the map's geometry.

In this article, we describe a digital method for the analysis and visualization of the planimetric accuracy of old maps. The visualizations are based on a series of geometrical transformations that are applied to control points identified on an old and a new reference map. The results are graphical representations of map distortion, as well as statistical and geodetic measures describing the map geometry (e.g., map scale, rotation angle, and map projection). The method presented is based on point locations that can be identified on the old map and the reference map. It does not directly compare the geometry of selected areal or linear features, such as the geometric accuracy of coastlines [11,12] or the locations and geometry of rivers [13,14].

Our method is based on previous work by various authors. Tobler [15,16] introduced bidimensional regression as a means of comparing the degree of resemblance between two sets of points, and illustrated the method by comparing locations identified on a 14th century map of the British Isles with their actual positions. He pioneered the visualization of geometric strains in historical maps using digitally generated warped grids, Tissot's indicatrix, displacement vectors, and vector fields. Boutoura and Livieratos [17] and Tobler [18] suggested the use of a homogeneous transformation to bring the coordinate systems of two maps into coincidence, before computing geometric differences between the maps. Beineke [19] suggested an improved method for the generation of distortion grids based on multiquadric interpolation introduced by Hardy [20]. Mesenburg [12,21,22] and Tobler [18] developed methods for the computer-assisted establishment of projections of old maps.

We build on these previous works in two ways. Firstly, a planimetric analysis is formalized as a series of geometric transformations that bring the coordinate systems of the two maps into coincidence. The central piece of this transformation sequence is a homogeneous transformation. We alternatively replace it with an affine transformation, which is useful for detecting and compensating the influence of non-homogeneous shrinking or shearing of the drawing support. Secondly, an important link in the chain of transformations is the reprojection of the reference points to compensate for the influence of the map projection of the reference map. The goal is to avoid artificial distortion patterns induced by the map projection of the reference map. If the projection type or the associated parameters are unknown, the projection is estimated.

## 2. Converting between coordinate systems

The general procedure for a cartometric analysis of a map is to compare an old map with a modern georeferenced map (i.e. metrically referenced) with a known geodetic coordinate system, which includes a geodetic datum – specifying the shape and position of a reference ellipsoid – and a cartographic projection. Two sets of corresponding points are used. One set originates from the modern reference map and is considered to be perfectly accurate, while the points of the old map are supposed to be inaccurate. The two sets of control points are used to bring two maps into coincidence for further analysis and extraction of historical data stored in the map. A sequence of geometric transformations is applied to the control points to register the old map with the coordinate system of the reference map.

### 2.1. The central Euclidean or affine transformation

The central link of the transformation sequence is a homogeneous or an affine transformation determined by a least squares estimation. A four-parameter Euclidean transformation (also called Helmert transformation in geodesy) with a uniform scale factor, a single rotation, and two offsets is the default suggested, as a Euclidean transformation does not shear or unilaterally distort the geometric space defined by the old map. Affine transformations may be used when considerable shearing or unilateral distortion of the drawing support are to be compensated; a five-parameter transformation allows independent scaling in two directions at right angles, which may be useful for compensating non-homogeneous paper shrinking, while a six-parameters transformation adds an additional rotation to remove geometric shearing.

The least-squares method determines the transformation by globally minimizing the squared distances between the two point

sets. The least-squares method for computing the parameters of a Euclidean transformation is discussed here, but the method can be applied in a similar way to affine transformations with five or six parameters. The linear system expressing a Euclidean transformation with four unknown parameters  $a_i$  is

$$\begin{aligned} X_i &= a_1 + a_3x_i - a_4y_i \\ Y_i &= a_2 + a_3y_i + a_4x_i \end{aligned} \quad (1)$$

where  $x_i$  and  $y_i$  are the horizontal and vertical coordinates of the points in the reference map, and  $X_i$  and  $Y_i$  are the coordinates of the target points in the historical map. In matrix notation, the system of linear equations can be expressed as

$$l = Aa \quad (2)$$

with

$$l = [X_1 \dots X_n Y_1 \dots Y_n]^T$$

$$A = \begin{bmatrix} 1 & 0 & x_1 & -y_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & x_n & -y_n \\ 0 & 1 & y_1 & x_1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & y_n & x_n \end{bmatrix}$$

$$a = [a_1, a_2, a_3, a_4]^T \quad (3)$$

The transformation parameters  $\hat{a}$  can be estimated with the standard least-squares method

$$\hat{a} = (A^T A)^{-1} A^T l \quad (4)$$

From  $\hat{a}$  the global rotation angle  $\alpha$  and the scale factor  $m$  of the old map are

$$\begin{aligned} \alpha &= \arctan(a_4/a_3) \\ m &= \text{sqrt}(a_3^2 + a_4^2) \end{aligned} \quad (5)$$

To estimate the mean error of the coordinates,  $s_0$  is computed, a quantity that we call standard deviation in this context

$$s_0 = \sqrt{\frac{\mathbf{v}^T \mathbf{v}}{2n_p - 4}} \quad (6)$$

where  $n_p$  is the number of control points and  $\mathbf{v}$  is the vector of distances for each pair of control points (4 is subtracted in the denominator as four parameters are estimated for a Euclidian transformation).

The standard deviations  $s_m$  and  $s_\alpha$  indicate the accuracy of the global scale factor  $m$  and the rotation angle  $\alpha$

$$\begin{aligned} s_m &= s_0 \sqrt{q_{33}} \\ s_\alpha &= (s_0 \sqrt{q_{33}}) / m \end{aligned} \quad (7)$$

with  $q_{33}$  as the third element on the diagonal of the matrix  $(A^T A)^{-1}$ .

A comparison of the standard deviations of the three transformations with four, five, or six parameters can be useful for determining a unilateral shrinking or shearing of a map due to an unstable drawing support. For example, if all three transformations have similar standard deviations, unilateral shrinking or

stretching is unlikely. However, if the five-parameters transformation has both clearly distinct scale factors and a smaller standard deviation, shrinking could possibly have occurred.

The method can be extended by introducing a weight matrix with an individual weight for each point. The weight of a point is proportional to its estimated reliability, that is, smaller weights are assigned to less precisely mapped features (e.g., mountain peaks) and more precise features (e.g., populated places) receive larger weights. The weight factors are added to the diagonal matrix  $P$  by adjusting the values on its diagonal, and Eq. (4) becomes

$$\hat{a} = (A^T P A)^{-1} A^T P l \quad (8)$$

A further enhancement is the inclusion of robust estimation that reduces the influence of aberrant control points, which are relatively frequent in old maps. Beineke [19] explores the use of standard robust estimators, such as the Huber [23] and the Hampel estimator [24], and presents a new estimator – called V-estimator – particularly developed for the analysis of old maps.

### 2.2. Considering the projection of the old map

For the computation of accuracy visualizations, it is essential that the old map and the reference map share a common map projection. Additional strains would be introduced otherwise, and distortion patterns stemming from a difference in projections would falsify the accuracy visualizations. In the general case, however, the old map and the new reference map do not share a common projection or geodetic coordinate system. Control points are therefore transformed from the reference map to the projection of the old map. This transformation is done via a so-called inverse projection, transforming the control points from the projected coordinate system to spherical longitude and latitude positions, followed by a forward projection that converts the points to the coordinate system of the old map (Fig. 1). This procedure is a standard functionality of Geographic Information Systems.

For the study of old maps, the influence of the geodetic coordinate system (i.e. the shape and position of the reference ellipsoid) can often be neglected, as the influence of mismatching geodetic coordinate systems is often small compared to the inherent planimetric distortions of the map. When analyzing a relatively precise map, however, geodetic methods exist to transform between geodetic coordinate systems.

The procedure outlined above assumes that the projection of the old map is known. However, the projection or the projection parameters for the old map is often unknown or not known with certainty. The old map's projection can be found by a trial-and-error method, where a series of candidate projections are evaluated [12,18,22,25]. For each candidate projection, the control points in the reference map are converted to the old map (Fig. 1). The points are first converted to spherical longitude and latitude coordinates using an inverse map projection (the leftmost transformation in Fig. 1), then forward projected with the candidate projection, and finally transformed using a Euclidean or affine

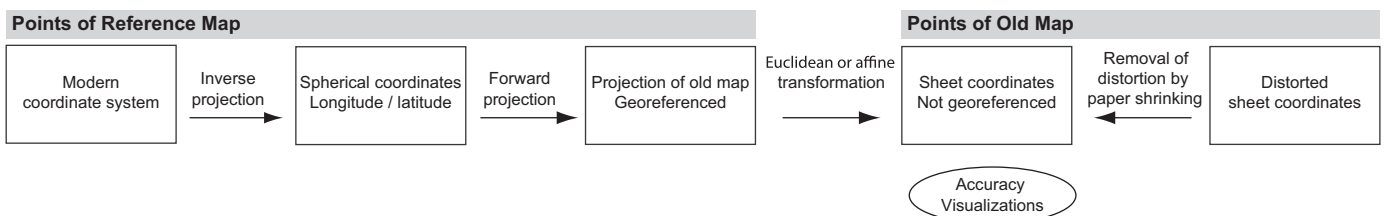


Fig. 1. Transformation sequence for generating accuracy visualizations in the old map. Each box represents a coordinate system, each arrow a transformation.

**Table 1**  
Selected map projections used during the Renaissance (after Snyder [27]).

Modern name	Year of invention
Equiangular	ca. A.D. 100
Trapezoidal	ca. 150 B.C. (?); A.D. 1426
Ptolemy's first	ca. A.D. 150
Ptolemy's second	ca. A.D. 150
Globular projections with the entire sphere in a circle by Bacon, Apian (I and II), Nicolosi	ca. A.D. 1265 (Bacon), ca. 1524 (Apian), 1660 (Nicolosi)
Orthographic	ca. 150 B.C.
Gnomonic	ca. 550 B.C.
Stereographic	ca. 150 B.C.
Polar azimuthal	B.C. (?); A.D. 1426
Simple or equidistant conic	1506
Werner cordiform	ca. 1500
Oval	ca. 1510
Mercator	1569
Sinusoidal, Sanson-Flamsteed	1570

**Table 2**  
Selected map projections developed during 1670–1799 (after Snyder [27]).

Modern name	Year of invention
Cassini	1745
Equidistant conic	1745 (Delisle)
Bonne	1752
Murdoch	1758
Lambert conformal, and equal-area conic	1772
Lagrange	1772 (Lambert)
Transverse Mercator	1772 (Lambert)
(Transverse) Cylindrical equal area	1772 (Lambert)
Lambert azimuthal equal-area	1772
Euler	1777

transformation determined with the least-squares method, as described in Section 2.1. Finally the standard deviation for the distances between pairs of control points is computed in the sheet coordinate system of the old map. This procedure is executed with various types of candidate projections and with varying projection parameters, altering the parameters step wise within predefined ranges. The type of parameters varies with the projection tested (e.g., central meridian of projection, latitude of true scale for cylindrical projection, latitude of tangency or two secant latitudes for conic projections, etc.). The projection with the smallest standard deviation of the distances is then used for the final transformation of the control points. This procedure for determining an unknown projection from a series of candidates has been successfully tested on synthetic point sets, maps of known projections, and maps where the projection could be determined by a complementary analysis of the graticule's geometry [26].

The development of map projections and their use in the past is thoroughly described by Snyder [27], who compiled historical projections, ordered by year of invention and application. Table 1 presents the most important projections used during the Renaissance, and Table 2 lists the most important map projections developed until the end of the 18th century. Literally hundreds of projections have been developed since then. Compilations of map projections developed for later periods can be found in Snyder's standard reference [27]. Many of these projections are not commonly used anymore in current cartography, and not all are mathematically well defined. The compilation in Tables 1 and 2 is nevertheless an important help for identifying candidate projections for early maps with the described trial-and-error method.

### 2.3. Compensating the effect of paper shrinking

In some cases the sequence of transformations described so far can be extended with an optional transformation to compensate for unilateral map shrinking. In Fig. 1, this is the rightmost transformation represented. Paper, vellum and other drawing supports are not inert materials and often shrink or dilate in a non-isotropic way. Maps with a well defined regular grid or other regular geometric structures (as for example the circularly radiating lines on Portolan charts for seafaring first made in the 1300s) can be used to compensate for the deformation introduced by inert drawing supports [12, 28, (cited in 29)]. However, this transformation is rarely done, and such methods have not been analyzed systematically.

## 3. Visualizing the geometric accuracy

Graphic representations of the planimetric accuracy allow for assessing the spatial variation of the geometric accuracy of a map, as well as for discovering and understanding new facts about a map. They are also an excellent means to illustrate these findings. It is thus essential to have a range of complementary visualizations available that illustrate the local geometric characteristics of the map. Map historians have developed various methods for analyzing and visualizing distortion in historical maps (see Forstner and Oehrlri [30] and Livieratos [31] for overviews). Almost all visualizations derive from two sets of corresponding points, i.e. one set in a modern reference map and one in the old map. The transformation sequence described in the previous section is a prerequisite to bring the coordinate systems of the two maps into concordance.

### 3.1. Displacement vectors

Displacement vectors are algorithmically a simple visualization technique and are easy to comprehend (Fig. 2). Each vector line starts at a point that was previously identified in the old map and ends at the position where the point would be if the old map was as accurate as the modern reference map, i.e. the vector lines end at the control points that are transformed from the reference map to the old map, using the transformation sequence in Section 2. Exceedingly long vectors are easily identifiable and indicate outliers that are due to gross positional errors in the map. Patterns of locally homogenous vector orientation and length become easily visible.

### 3.2. Distortion grids

The rotated, compressed, or enlarged meshes of a distortion grid reflect the local deformation and rotation of the old map (Fig. 3). Map historians have used deformed grids to visualize map distortion for a long time. Wagner [33] might have been the first in 1895 to have constructed a grid for a historical map. When manually constructing a distortion grid, the map historian fits grid lines into a field of reference points based on rather subjective estimations [34]. Computer-based construction objectives and accelerates the rather tedious manual construction.

To construct distortion grids, we apply a method developed by Beineke [19], which is based on the multiquadric interpolation introduced by Hardy [20]. This method offers the advantage of minimizing the influence of points with gross errors and prevents the generation of closed circular lines. It is important to note that other authors have used alternative computer-assisted methods, which produce distortion grids that differ in shape, for example interpolation in triangulated networks, variations of bidimensional regression [16], digital simulation of a manual construction technique [35], or distance weighting [35].

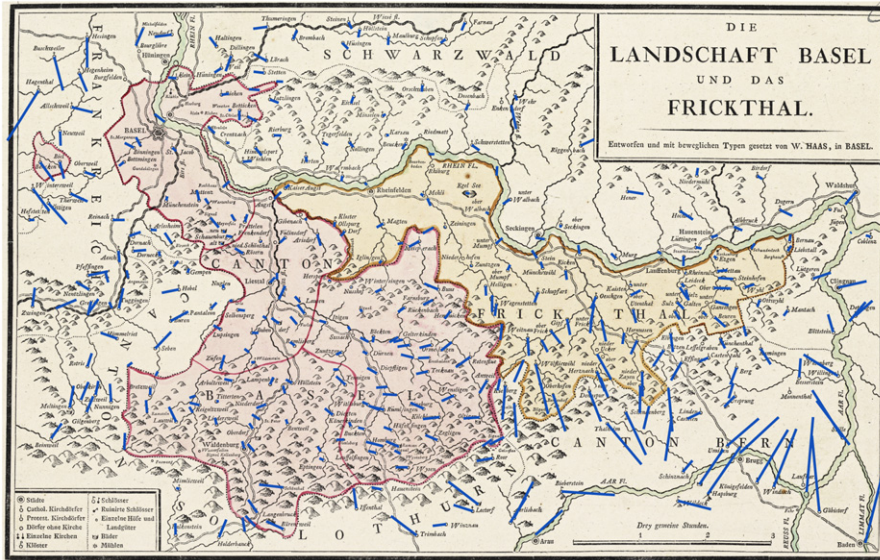


Fig. 2. Displacement vectors connecting mapped positions with their true location. Map: [32].

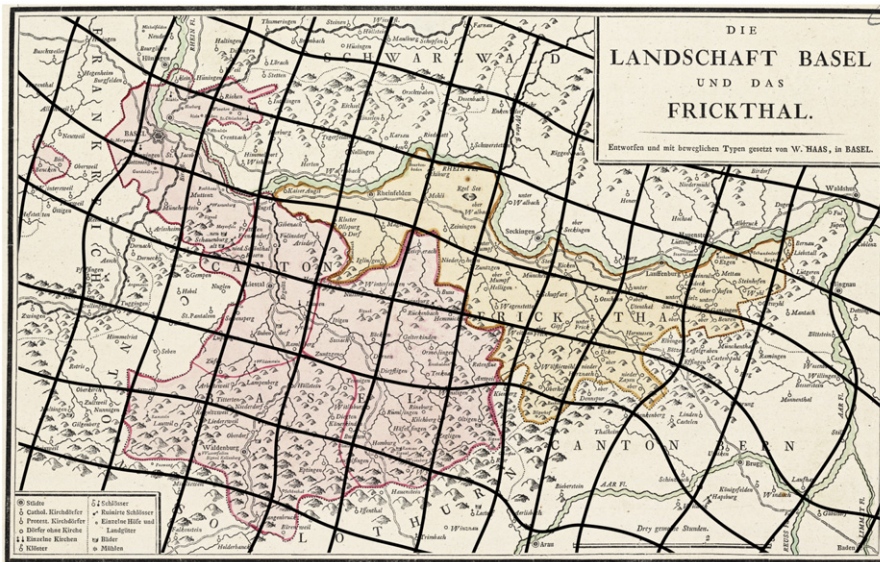


Fig. 3. Distortion grid with a cell size of 5000 m for an old map [32].

With Beineke's method [19], the distortion grid is constructed as follows: first, displacement vectors on the old map are computed by transforming the control points of the reference map to the old map, as described in Section 3.1. After the transformation, the horizontal and vertical differences  $u_i$  and  $v_i$  between the two sets of control points are computed.  $u_i$  and  $v_i$  are then used to determine the parameters of the multiquadric interpolation

$$\begin{cases} u_1 = d_{11}a_1 + d_{12}a_2 + d_{13}a_3 + \dots + d_{1n}a_n \\ u_2 = d_{21}a_1 + d_{22}a_2 + d_{23}a_3 + \dots + d_{2n}a_n \\ \vdots \\ u_n = d_{n1}a_1 + d_{n2}a_2 + d_{n3}a_3 + \dots + d_{nn}a_n \end{cases} \quad (9)$$

$$\begin{cases} v_1 = d_{11}b_1 + d_{12}b_2 + d_{13}b_3 + \dots + d_{1n}b_n \\ v_2 = d_{21}b_1 + d_{22}b_2 + d_{23}b_3 + \dots + d_{2n}b_n \\ \vdots \\ v_n = d_{n1}b_1 + d_{n2}b_2 + d_{n3}b_3 + \dots + d_{nn}b_n \end{cases}$$

with  $a_i$  an  $b_i$  as the unknown transformation parameters, and  $d_{ij}$  as the Euclidian distance between control points  $p_i$  and  $p_j$ . This

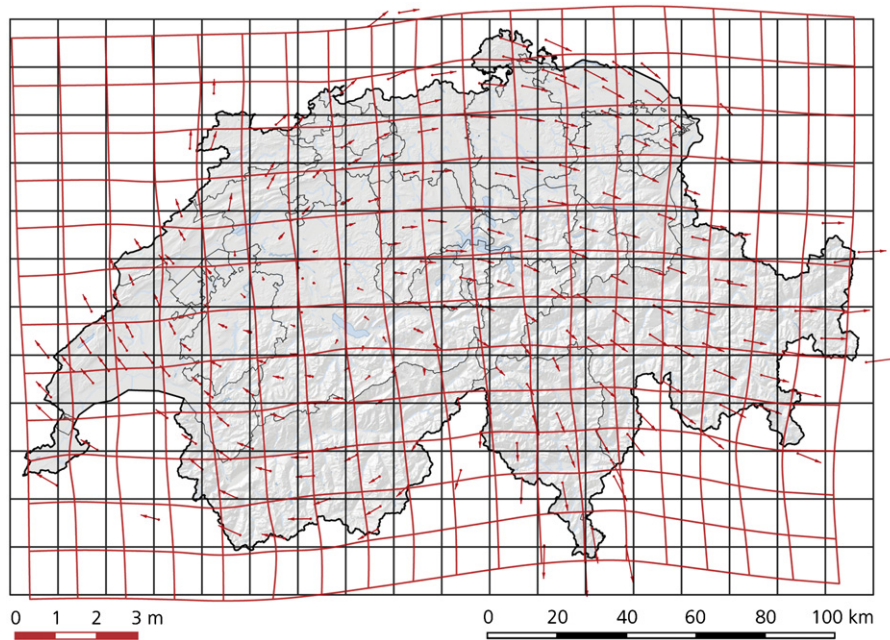
system of linear equations is simple to solve with Gaussian elimination.

The next step consists of the construction of a regular grid consisting of straight lines in the coordinate system of the reference map. The vertices are re-projected (Section 2.2) and the central transformation (Section 2.1) applied, which results in a regular, but scaled, rotated, and possibly sheared grid in the coordinate system of the old map. The grid is then deformed by adding the corresponding  $u_{vi}$  and  $v_{vi}$  displacements to each vertex, with  $u_{vi}$  and  $v_{vi}$  computed with the multiquadric interpolation for the grid vertex  $P$

$$\begin{aligned} u_{vi} &= \text{sum}(a_i s_i) \\ v_{vi} &= \text{sum}(b_i s_i) \end{aligned} \quad (10)$$

with  $s_i$  as the Euclidian distance between  $P$  and the control point  $p_i$ .

Apart from deformed Cartesian grids, distorted networks of lines of latitude and longitude can also be generated using this approach. These graticules are preferable for maps at small scales that depict entire countries, continents or the whole planet, as the bending of the parallels and meridians illustrates the projection of



**Fig. 4.** Exaggerated distortion grid (in red) illustrating the differences between two geodetic reference systems of a few meters only, combined with a regular reference grid (in gray) (from [36]). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the old map. To generate such a network of meridians and parallels, a graticule with regularly spaced vertices is first generated in longitude and latitude coordinates in the spherical coordinates system (Fig. 1). The vertices are then transformed with the forward projection, and the deformation procedure as outlined above is applied to the vertices of the grid.

When analyzing the geometric properties of relatively accurate maps, local distortions are difficult to detect visually, because grids consist of almost straight lines. The same problem also occurs when visualizing small movements or displacements originating from areas other than old distorted maps. Fig. 4 shows an example visualizing the difference between two geodetic reference systems. Local displacements in this data set are generally below a few meters, which would result in a visually regular grid, as the displacements are minuscule compared to the area covered by the map. Two methods have been applied in Fig. 4 to better visualize the small distortions: (a) an undistorted grid was combined with the deformed grid and (b) the distortions were amplified before computing the deformed grid.

The additional undistorted grid in Fig. 4 has the same dimensions and cell size as the distorted grid, and is scaled and rotated to optimally align with the control points, but consists of perfectly straight lines. The undistorted grid is in fact an intermediate product when computing the distorted grid. It is the grid of the reference map transformed to the old map after the re-projection (Section 2.2) and Euclidean or affine transformation (Section 2.1), but without the multiquadric interpolation applied. The second alternative is to amplify the distortions before the grid is deformed by scaling the distances  $u_i$  and  $v_i$  in Eq. (9) by a factor larger than 1. The result is a grid with exaggerated undulations.

#### 4. Visualizations in the reference map

The visualizations presented in the previous section are generated in the coordinate system of the old map, which is a local map sheet coordinate system. The transformation sequence outlined in Section 2 transforms points from the reference map to the old map and the methods in Section 3 create accuracy

visualizations displayed in the old map. This is not useful if the distortions of two or more old maps are to be compared, as they do not generally share a common projection. Neither is an accuracy visualization in the old map helpful if the old map is georectified, i.e. deformed and resampled such that it aligns with a modern reference system. Georectification is a required pre-processing step before information can be vectorized from the old map for use in a GIS. We therefore need accuracy visualizations in the coordinate system of the reference map.

For generating visualizations in the reference map, the direction of the Euclidean or affine transformation is inverted (see the direction of the bold arrow in Fig. 5). The visualizations are computed in the georeferenced coordinate system of the old map and then transformed to the reference map, applying an inverse and a forward map projection (Fig. 5 lower row). The numerical and graphical results (e.g., distortion grids) vary considerably depending on whether they are generated for the reference map or the old map. Figs. 6 and 7 showing a distortion grid and accuracy circles illustrate this for the London Underground map diagram. (Accuracy circles are the length of the displacement vectors as described in Section 3.1, represented as area-proportional point symbols.)

Combining the visualizations with the reference map is particularly useful for comparing multiple maps by combining their accuracy visualization with a reference map, as demonstrated in a case study in the next section.

#### 5. An application: determining a source map

To illustrate the concepts presented above, three historical maps are compared. The goal is to find out whether a manuscript map [37] of a part of northeastern Switzerland of around 1798 by Hassler (Fig. 8) is:

- (a) a copy of an older map by Nötzli [38] (an undated manuscript at a scale of approx. 1:110 000 drawn during the first half of the 18th century, showing the Swiss Canton Thurgau and its environs), or

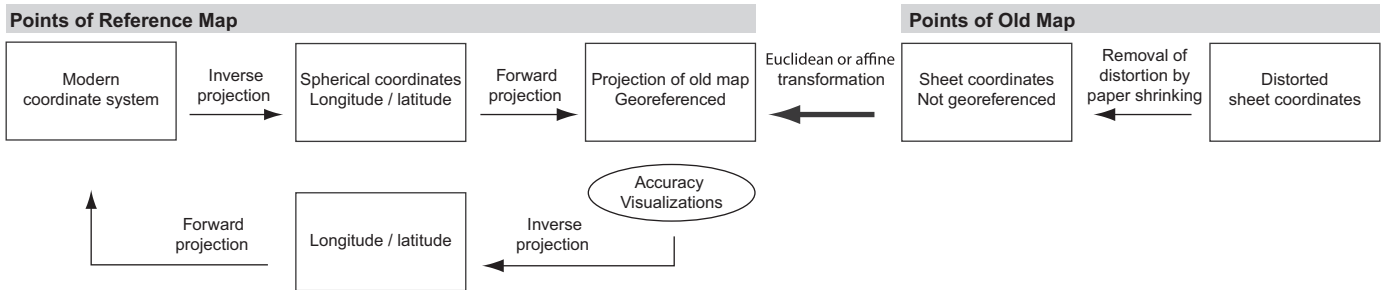


Fig. 5. Transformation sequence for generating visualization in the reference map. Each box represents a coordinate system, each arrow a transformation.

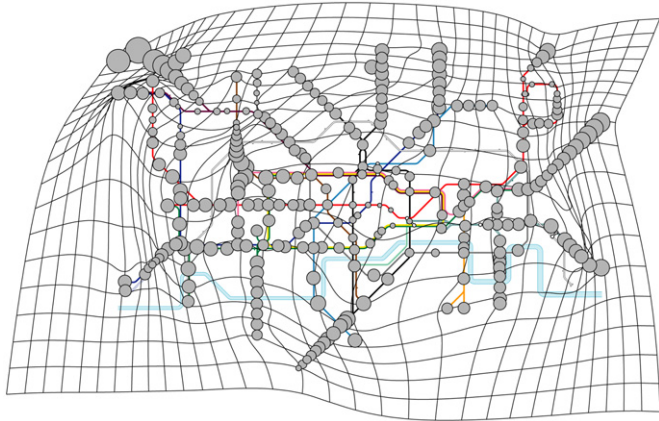


Fig. 6. Distortion grid and accuracy circles for the current London Underground map in the coordinate system of the map analyzed.

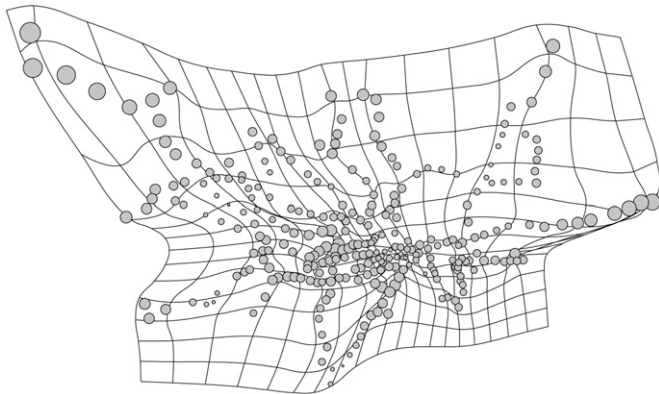


Fig. 7. Distortion grid and accuracy circles in the reference map represented in an undeformed Mercator projection.

- (b) a copy of the more recent Atlas Suisse [39] (the first nationwide mapping based on geodetic surveying published at the same time at a scale of approx. 1:120 000 in 16 sheets), or  
 (c) based on an unknown local surveying campaign.

Ferdinand Rudolf Hassler, the author of the studied map, was a surveyor, mathematician, and physicist, who emigrated to the US in 1805, where he headed the United States Coast Survey and the Bureau of Weights and Measures. Hassler's career and more details about the presented case study are described by Rickenbacher in [40].

Hassler's map was studied in 1879 by Wolf [41], who suspected it to be a copy of Nötzli's map. A comparison of the distortion grids of the three maps confirms this assumption. Figs. 9 and 10 show the grid for Hassler's map combined with the grids of the two maps that could have been used as a master

copy. Fig. 9 shows two clearly distinct grids with different distortion patterns, while Fig. 10 shows almost matching grids. For a more detailed analysis, we generate displacement vectors in the coordinate system of the modern reference map and combine the three data sets with a GIS (Fig. 11). The displacement patterns can be analyzed for each control point, and it becomes clear that the geometry of Hassler's and Nötzli's map are identical. The few differences are most probably due to inaccuracies by Hassler, and not based on an improved survey of the area. On the other hand, the displacement vectors of the Atlas Suisse differ considerably from the two other maps. Inaccuracies are at many places considerably larger and follow a different distribution pattern. Hence, it seems likely that Hassler copied Nötzli's map as assumed by Wolf, or – although considerably less likely – that both copied their maps from an unknown source independently.

## 6. Implementation

The presented method was implemented in MapAnalyst, an open-source Java application [42,43]. It allows for the efficient identification and management of control points both in the historical map and in the corresponding reference map. It computes distortion grids, error vectors, and isolines of scale and rotation. It also derives the historical map's scale, rotation angle, and statistical indicators from the central transformation (Eqs. (5) and (6)). MapAnalyst offers interactive tools to explore local variations of displacements, scale, and rotation, and supports a series of historical projections. It also includes the described method for determining unknown projections, provides various types of transformations (including robust estimators), and integrates OpenStreetMap (a free map of the world [44]) as a ready-to-use reference map.

## 7. Discussion and conclusion

For assessing the topographic accuracy (i.e. the quantity and quality of information about represented landscape objects) expert knowledge about the specific map is required. Estimating the chronometric accuracy of an undated historical map is often difficult, requiring carto-historical and carto-bibliographic knowledge. The planimetric and geodetic accuracy, however, can be evaluated and visualized with the presented method. The cartometric analysis of historical maps is undoubtedly faster and more reliable with digital methods than with traditional manual techniques. MapAnalyst and the method described were well received among historians, map historians, and geographers [45], and were used for a series of studies [26,29,46–66]. Current limitations of MapAnalyst are slow display performance with large raster images (e.g., more than 100 megapixels), missing



Fig. 8. Overview and section of the manuscript map "Carte vom Thurgau" by Hassler [37].

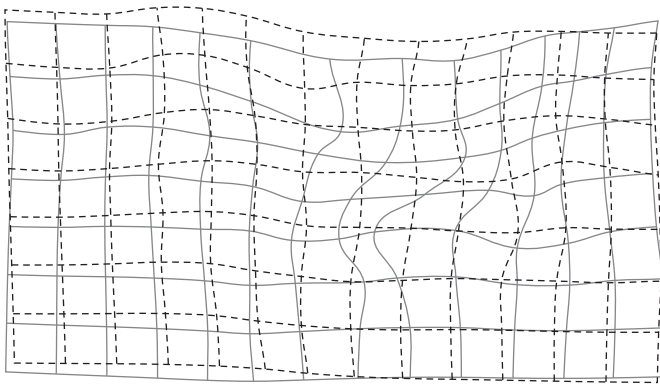
rectification functionality for raster images, and the limitation to early projections developed before the end of the 18th century.

Many transformation methods between coordinate systems have been developed in geodesy, computer graphics, and other fields. The procedure described here is therefore not the only possible method. However, it offers certain advantages compared to other approaches. For example, the scale of the old map can be derived from a Euclidean or affine transformation, which is considerably more accurate and reliable than methods based on the measurement of the length of a few selected reference lines. Also, unilateral deformation due to paper shrinking can be

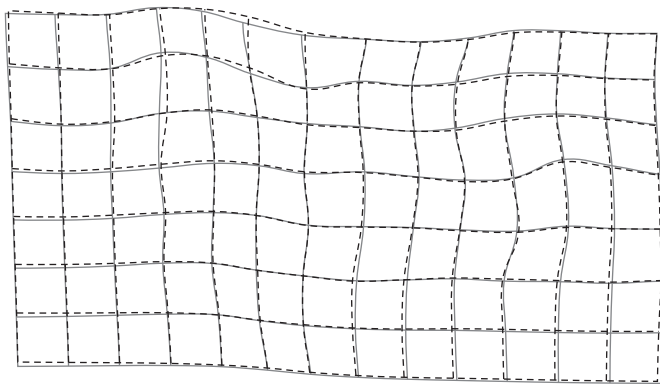
compensated with an affine transformation, and projections can be taken into account.

For maps at large scales (i.e. covering a small geographic area), the projection does not play an important role and its influence on the map geometry can often be ignored. This is also the case for maps at medium scales that have considerable distortion. For maps at small scales, however, the projection should be taken into account by transforming the reference points to the coordinate system of the old map. For the frequent cases where the map projection is unknown, a search method is presented in this paper. Still, the historian must scrutinize the projection suggested by the

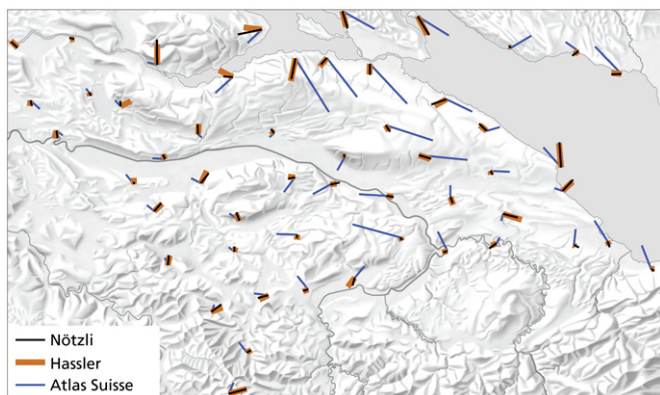




**Fig. 9.** Diverging distortion grids for Hassler's map (dashed lines) and for the Atlas Suisse, the first potential source map (solid lines).



**Fig. 10.** Coinciding distortion grids for Hassler's map (dashed lines) and for the second potential source map by Nötzli (solid lines).



**Fig. 11.** The displacement vectors of Hassler's and Nötzli's maps coincide, the vectors of the Atlas Suisse are spatially unrelated.

method to ensure that the projection was already known and applied when the map was drawn. The method may also suggest illogical projection parameters if not configured properly. For example, a central meridian for a conic projection may be suggested that is off the central area covered by the examined map.

The transformation sequence presented in this paper can be used to derive various types of graphical representation of map deformation. We presented distortion grids, displacement vectors, and accuracy circles. Alternative types of representations and numerical methods continue to be developed [30,31]. Examples include distortion ellipses [16], isolines of local map scale

factor and rotation angles [43], error roses [64], and gridded area errors [67]. Such visualizations are not limited to the analysis of old maps. For example, distortion grids generated with MapAnalyst were used by Reimer and Dransch [68] to illustrate the geometry of choromatic maps (generally strongly schematized maps), and by Kistler et al. [36] for visualizing strain in a geodetic reference system. Further areas of applications are visualizations in comparative anatomy for visualizing the evolution of skeletons (e.g., in [69]).

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