SHORT ARTICLE

Automated Reduction of Visual Complexity in Small-Scale Relief Shading

Anna M. Leonowicz
Institute of Cartography / ETH Zurich / Switzerland
Department of Geography / University of Zurich / Switzerland

Bernhard Jenny, and Lorenz Hurni
Institute of Cartography / ETH Zurich / Switzerland

Abstract

Shaded relief derived automatically from digital elevation models differs distinctly from traditional manual shading. Particularly at small scales, many small topographic details that are present in terrain models disturb the clear portrayal of the main relief features. Automatic shading is therefore not appropriate for high-quality cartographic products. This paper proposes a new method of generalizing digital elevation models for deriving small-scale shaded relief that resembles the manual style. The procedure consists of the following raster operations: undesirable topographic details are smoothed with low-pass filters, and the main landforms, such as ridgelines and valleys, are detected by curvature coefficients. Two secondary grids are derived, one exaggerating ridgelines, the other deepening valley bottoms, and the two grids are combined according to the character of the terrain; the grid with exaggerated ridgelines is used in mountainous areas, and the grid with deepened valley bottoms in lowland areas. Finally, shaded relief is derived from the combined elevation model. Following these processing steps, only a few manual corrections are necessary to produce high-quality small-scale relief shading.

Keywords: shaded relief, hill shading, digital elevation models, generalization, small-scale maps, curvature-based shading

Résumé

L'estompage automatique des modèles d'élévations numériques diffère grandement de l'estompage manuel traditionnel. Plus particulièrement, à petite échelle, de nombreux petits détails topographiques qui sont présents dans les modèles numériques de terrain empêchent la représentation claire des principales caractéristiques du relief. Par conséquent, l'estompage automatique n'est pas adéquat pour les produits cartographiques de haute qualité. Dans l'article, on propose une nouvelle méthode de généralisation des modèles d'élévations numériques pour obtenir des estompages à petite échelle qui ressemblent au style manuel. La procédure consiste à ajuster les détails topographiques indésirables à l'aide de filtres passe-bas et à détecter le relief principal, comme les crêtes et les vallées, avec des coefficients de courbure. Deux grilles secondaires sont dérivées : l'une exagère les lignes des crêtes, et l'autre accentue le fond des vallées. Puis on combine les deux grilles selon les caractéristiques du terrain. On utilise la grille avec les crêtes exagérées pour les régions montagneuses, et celle avec les vallées accentuées pour les plaines. Enfin, l'estompage est obtenu à partir du modèle d'élévation combiné. Après ces étapes de traitement, on apporte quelques corrections manuelles pour produire un estompage à petite échelle de grande qualité.

Mots clés : estompage automatique, estompage manuel, modèles d'élévations numériques, généralisation, carte à petite échelle, estompage basé sur la courbure

doi: 10.3138/carto.45.1.64

1. Introduction

Shaded relief maps were traditionally created by a few artistically skilled cartographers with good insight into relief interpretation. Today, manual skills are no longer necessary, as shaded relief is automatically computed from digital elevation models. Shading algorithms are available in many GIS applications and raster graphic software. Automatic relief shading (i.e., analytical shading) has many advantages over manual production: it is faster and less expensive, does not depend on the individual style of a cartographer, and is rich in terrain details. However, the functionality offered by standard computer software does not always meet the needs of cartographic relief presentation. Analytical shading depicts relief details with photographic quality, but it fails to emphasize the typical landforms that characterize the specific relief types. Adjustments of local light direction, aerial perspective, and a uniform grey tone on flat areas are used for manual shading in order to graphically enhance the three-dimensional relief impression. Standard automatic shading functionalities do not implement such operations. Many cartographers today have little or no experience of manual relief shading and consider automated relief shading the de facto standard. However, cartographers who have produced shaded relief using manual techniques, and who have specialized in this area, consider that a carefully executed manually shaded relief is still superior to its automatically generated equivalent.

One can find few references concerning the production of shaded relief for small-scale maps (i.e., maps at 1:500 000 or smaller[1]) in the cartographic literature. Eduard Imhof (1982) developed general guidelines for the generalization of shaded landforms, with examples at different map scales. The successive steps of elimination and exaggeration of terrain features, according to an increasing reduction in scale, are described as follows:

> First to be smoothed out or removed are the gullies, niches, projections, local gentle slopes, erosion terraces, and the small details of alluvial deposits over the ground. Next, whole valleys and mountain ridges are eliminated, and what was a complex mountain group with many valleys becomes what appears to be one mountain only. Narrow, but still orographically significant river valley grooves are stressed by the appropriate adjustment of light and shading tones … Finally, at the smallest scales, maps are composed almost exclusively of similar mountain chains, patterns of high mountain peaks and flat plains. (Imhof 1982, 188)

During generalization, cartographers try to maintain the character of landforms (e.g., the sharpness of ridges, the shape of valley profiles) and to accentuate the differences among relief types (e.g., high-alpine mountain relief, moderately hilly plateaus, low relief with big river valleys, coastal plains). Even if identical design principles and base data are used, results vary considerably among cartographers. This is illustrated by Figure 1, which shows the same geographical area, shaded by six different cartographers, based on the same data. Thus, the quality of manual relief is highly variable and depends greatly upon the skill, experience, and individual style of the relief artist, as well on the amount of time spent on creating it.

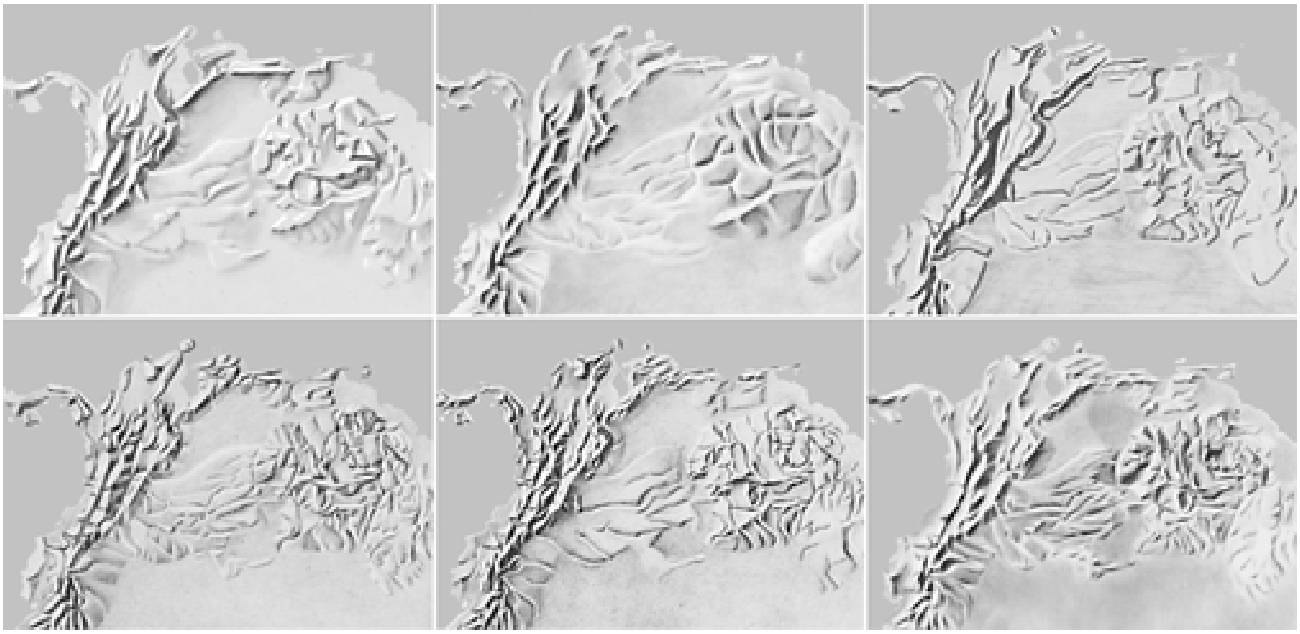

Figure 1. South America's relief, shaded by six different cartographers; © ETH Zurich, reproduced by permission.

2. Previous Digital Relief Shading

Diffuse reflection, introduced by Pinhas Yoeli (1959, 1965), was a first attempt at computer-generated shading. The value of grey shades is defined as a cosine of the angle between surface normal and the light direction. Since this first effort to calculate relief shading by computer, many advances have been made in the attempt to adjust analytical shading for specific cartographic needs. Various researchers have pursued the integration of the principles developed for traditional manual shading into digital technology. Several illumination models have been developed (these are thoroughly described and compared by Horn 1982). An alternative uniform illumination model was recently proposed by Patrick Kennelly and A. James Stewart (2006). Other researchers have focused on simulating local adjustments of the light direction (Yoeli 1967; Brassel 1974; Mark 1992; Jenny 2001; Loisios, Tzelepis, and Nakos 2007) and the application of aerial perspective (Brassel 1974; Jenny 2001; Patterson 2001a).

Automatic relief shading at small scales, however, has received little research attention so far, and the discrepancy in quality between analytical and manual shading is especially great in this area. Automatically produced shaded relief at small scales is excessively detailed, especially when it is derived from high-resolution sources. Highly detailed shading is often visually inefficient because the main landforms are not clearly visible. As Tom Patterson (2001b) notes, the dense details obscure macro topography: it is impossible to see the forest for the trees.

To reduce visual complexity, relief needs to be generalized by removing unwanted terrain details and accentuating important landforms. Various methods have already been proposed for generalizing digital elevation models, but only a few studies consider the specific needs of relief shading. Robert Weibel (1992) differentiates among three approaches for terrain generalization: global filtering, selective filtering, and heuristic methods. *Global filtering* computes a statistical measure for each output raster cell, for example, the mean or median value of neighbouring data values. *Selective filtering* removes insignificant elevation points (i.e., points that are below a predefined significance level). *Heuristic methods* take into account important topographic features (e.g., ridge and valley lines), which are preserved in the output elevation model. Only heuristic methods tend to follow the principles of cartographic generalization (Weibel 1992), and they are therefore the preferred solution for cartographic relief representations.

Two different approaches for generalizing shaded relief exist: generalizing the elevation model prior to calculating analytical shading, and filtering derived shaded relief. Böhm (1997, 2000) developed a digital elevation model (DEM) generalization technique that uses spatial filters and prevents valleys and ridgelines from being over-smoothed. Nikolas Prechtel (2000) produced small-scale relief shading using a customized re-sampling method and vertical exaggeration of the elevation values. Patterson (2001c, 2001d) developed a technique called "resolution bumping" that merges down-sampled elevation data with high-resolution data. Median filtering has also been used by Patterson (2001a) as a tool applied on a shaded relief (already derived from elevation data) that smoothes local irregularities and preserves sharp ridgelines and valley lines.

This paper presents a new method for the generalization of digital elevation models, specifically designed for calculating shaded relief with a greatly reduced level of detail. An efficient method for the automatic production of cartographically generalized shaded relief at small scales is highly desirable: relief shading, together with hypsometric tinting, is a preferred and commonly used relief representation at small scales, because it emphasizes the third dimension (Imhof 1982).

3. Curvature-Based Terrain Generalization

In developing our automatic generalization approach, we drew inspiration from the manually created relief shading shown in Figure 2. It shows terrain in the Swiss style, according to principles defined by Imhof (1982). We deliberately concentrate on this style, which is regarded as exemplary by many. For example, J.S. Keates finds that "the most sophisticated and elaborate representation is the Swiss, using contours . . . , detailed rock drawing and hill shading" (1996, 257). The reference shaded relief in Figure 2 shows the eastern part of Switzerland at a scale of 1:1,000,000. Accordingly, we concentrated on automating relief shading for mountainous areas. For purposes of comparison, Figure 3 depicts a shaded relief at the same scale that was calculated automatically using diffuse reflection (Yoeli 1959), with the light source at an azimuth of 300° and an inclination of 45°. It was derived from the Shuttle Radar Topography Mission (SRTM)[2] elevation model, which provides elevation data at a resolution of 3 arc seconds (approximately 60 m at the latitude of Central Europe). At a scale of 1:1,000,000, the high level of topographic detail present in the SRTM model is visually disturbing. In comparison with manual shading, mountain ridges are not sharp enough, large landform structures are not prominent, and small features of the low relief are almost invisible. From this example it is obvious that automatic shading does not differentiate sufficiently between relief types (in this case, alpine, lowland, and hilly landscapes).

In developing our curvature-based shading method, we focused on Imhof's (1982) design principles:

1. small-scale relief shading should visibly accentuate the differences between mountainous and low-relief areas

Figure 2. Manual relief shading: 1:1,000,000; © Federal Office of Topography swisstopo (pixel map: swisstopo DV033492.2), reproduced by permission.

2. irrelevant small terrain details should be removed
3. important mountain chains should be accentuated
4. main river valleys should be visually enhanced

The manual shading in Figure 2 was created according to these principles: ridgelines are mostly visible in the high mountains; the lowlands, on the other hand, look like flat plains carved by river valleys.

To follow these principles in our automatic approach, mountains and lowlands are generalized separately, in different ways, and are later recombined into one elevation model. The complete generalizing procedure consists of successive operations performed on digital elevation data; the procedure is outlined in Figure 4. The steps necessary to generalize lowland and mountain areas are indicated in grey. The operations shown on the right-hand side of the diagram produce an additional grid, which is used to combine lowlands and mountains into one generalized elevation model. Shaded relief is subsequently calculated from the generalized elevation data.

3.1 SPATIAL FILTERING

The first step of the generalizing procedure filters the digital elevation data with a low-pass mean filter, which computes a new pixel value by averaging the values within its immediate vicinity. Mean filters are frequently used for smoothing images (Burrough and McDonnell 2000) and are also applicable to the removal of details from digital elevation models. A mean filter of 5 × 5 pixels is applied 15 times to smooth the SRTM terrain at a spatial resolution of 3 arc seconds. The filter size and the number

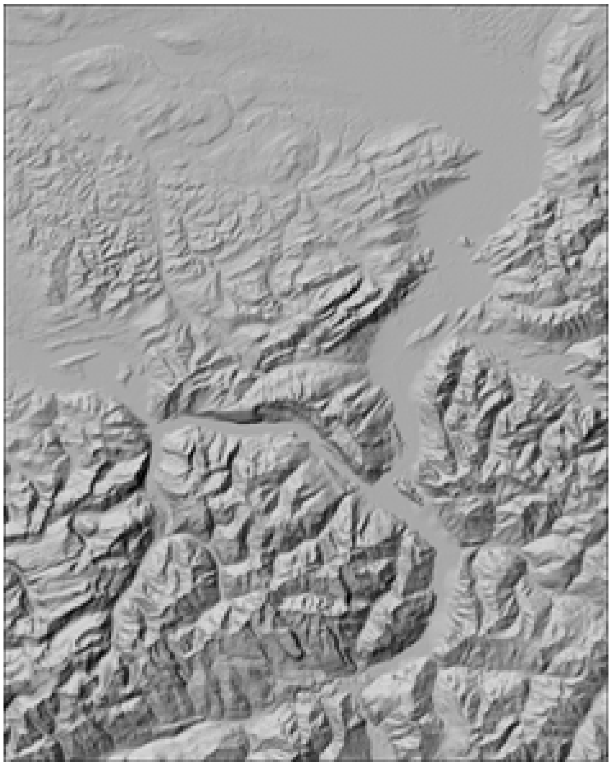

Figure 3. Diffuse reflection calculated from raw SRTM data.

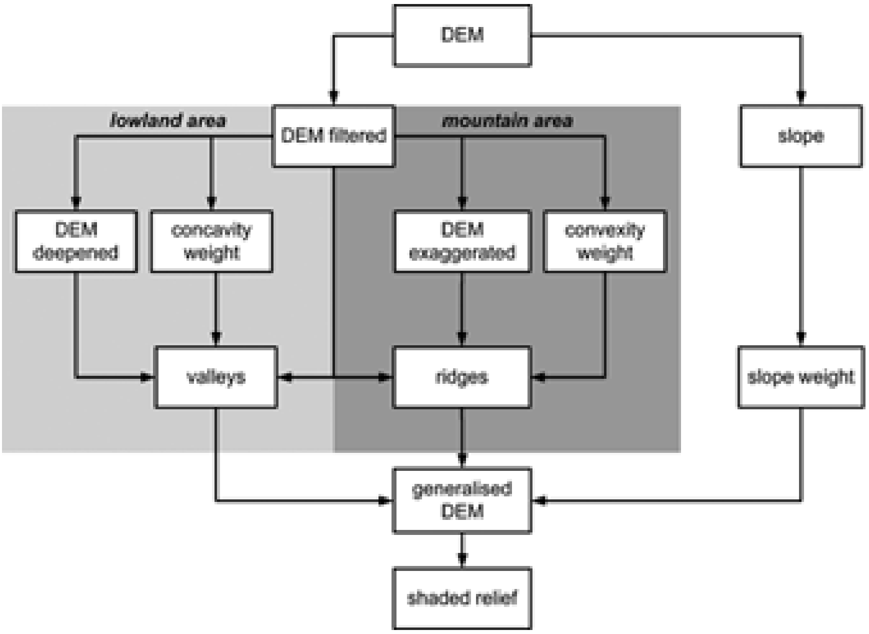

Figure 4. Processing steps leading to generalized relief shading

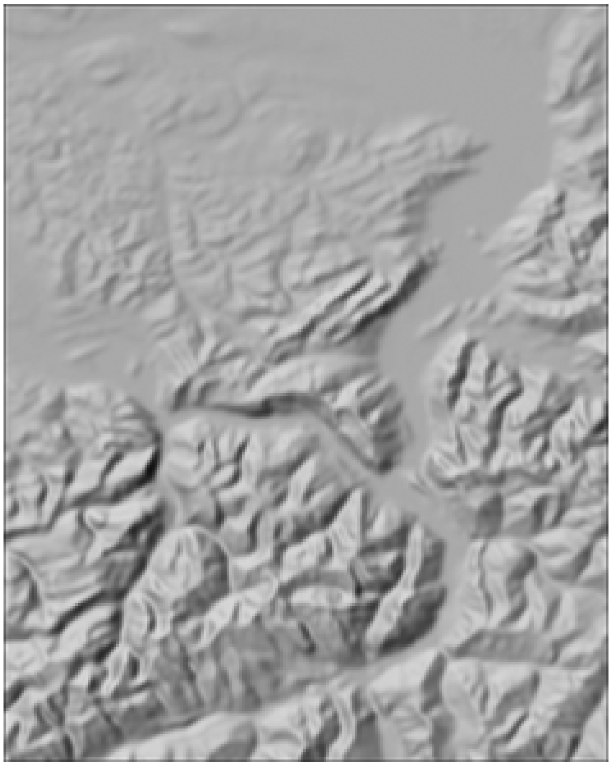

Figure 5. Relief shading derived from filtered SRTM data (mean filter, size 5 × 5 pixels, applied 15 times).

of filter runs depend on the map scale and the spatial resolution of the elevation model: a larger filter size and a higher number of filter passes result in stronger generalization. These parameters should be adjusted by visually inspecting the preliminary results to ensure that all unnecessary terrain details are removed from the elevation model. The shaded relief derived from the smoothed elevation model is shown in Figure 5: the relief is clearly over-generalized and is lacking sharp structures. These smoothed elevation data serve, in our procedure, as a greatly simplified base surface onto which the relevant details are added using the subsequent processing steps.

3.2 DETECTING TERRAIN FEATURES: RIDGES AND VALLEYS

Manual shading accentuates important relief features relative to analytical shading. Main ridgelines need to be stressed in mountainous areas, and main valleys in flat areas. Several techniques exist for detecting ridge- and valley lines from digital elevation models; these methods were developed in different research fields, especially in geomorphology (methods based on local terrain morphology) and hydrology (drainage direction). (For reviews of these methods see Tribe 1992; Bertolo 2000; Deng 2007; Minár and Evans 2008). Edge-detection techniques developed in the field of image processing (Gonzalez and Woods 2008) are also applicable here.

An approach based on local terrain morphology is used in our application; the terrain features are identified by curvature coefficients. A related technique that also uses curvature to enhance shaded relief has been developed by Kennelly (2008), but, in contrast to our method, this solution is aimed at large-scale mapping. Using plan and profile curvature, subtle terrain details (such as small changes in slope and drainage patterns) are highlighted on a shaded map, and such detailed visualization can support geomorphologic landscape analysis. Our procedure also uses curvature coefficients to enhance terrain features, but these are used in order to generalize relief (i.e., to remove small terrain details and enhance the most important landforms). For this purpose we use different coefficients of curvature: maximum and plan curvature to identify ridgelines, and minimum curvature to identify valleys. *Plan curvature* measures the rate of change of aspect along a contour line in the horizontal plane; it differentiates between convex and concave forms, and defines sharp and clear lines of ridges and valleys (Wilson and Gallant 2000). Coefficients of maximum and minimum curvature measure curvature in any possible plane (Wood 1996); they reflect the geometrical form of ridges and valleys, and this property is particularly useful in lowland areas, where ridges are not sharp and valleys have broad, flat bottoms. The curvature coefficients were calculated via the Evans–Young method, which uses a quadratic trend surface for each cell, fitted to the 3 × 3 local elevation matrix (the formulas used in this method are described in Pennock, Zebarth, and de Jong 1987; Wood 1996; and Shary, Sharaya, and Mitusov 2002). A comparison of the Evans-Young method with alternative, higher-order polynomial models (Schmidt, Evans, and Brinkmann 2003) has shown that it gives more stable results and is appropriate for use in modelling geomorphometric elements such as ridges and valleys.

Increasing the size of the local window, which is used for fitting the quadratic trend surface in the Evans-Young method, allows for detecting large terrain features and removing small details. This multi-scale quadratic parameterization is described by Jochen Schmidt and Robbie Andrew (2005) and by J.D. Wood (1996) and is implemented in LandSerf.[3] Another possibility is smoothing DEMs, using a mean filter before calculating parameters; this simple solution is proposed by Sanjay Rana (2006). In our procedure, the level of terrain detail is adjusted by selecting an appropriate window size for calculating the curvature coefficient.

3.3 EXAGGERATING RIDGES AND VALLEYS

The next processing step determines weights to vertically exaggerate ridges and valleys. Two weights are computed for each cell in the digital elevation model: one for exaggerating ridges, and the other for deepening valleys. The weight for deepening valleys is calculated from minimum

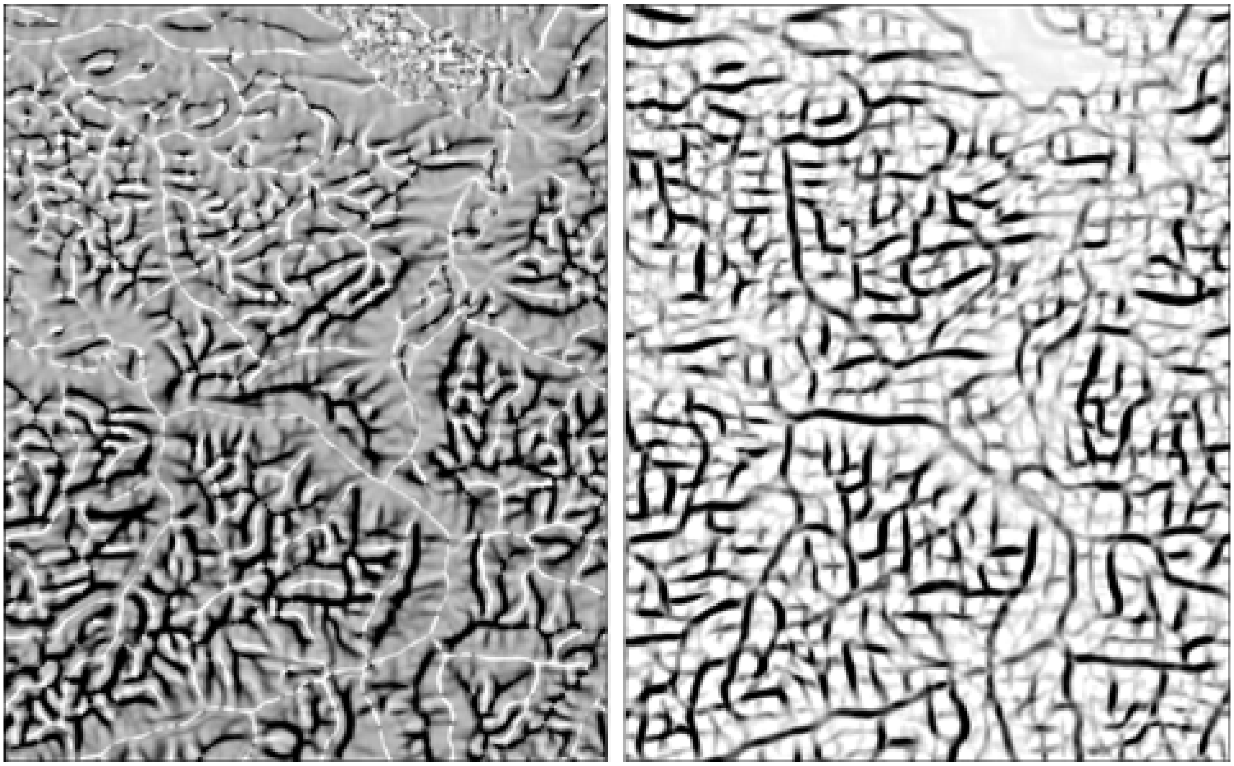

Figure 6. Weights for exaggerating ridges (left) and deepening valleys (right).

curvature; the weight for exaggerating ridges is calculated as a sum of maximum and plan curvature. Figure 6 shows both ridges' and valleys' weights: black indicates high values, and white indicates low values.

In the next step, two additional elevation models are generated by exaggerating and deepening the smoothed DEM produced by the step described in section 3.1 above. The exaggerated and the deepened models are calculated by multiplying the filtered model by a constant factor. Next, the exaggerated and deepened models are combined with the filtered model by curvature weights according to the equations (1) and (2), which are applied to each cell of the elevation model:

$$h_{mount} = w_r \cdot h_{ex} + (1 - w_r) \cdot h_{fl} \tag{1}$$

$$h_{low} = w_v \cdot h_{deep} + (1 - w_v) \cdot h_{fl} \tag{2}$$

where

h_{mount} = elevation of mountain DEM
h_{low} = elevation of lowland DEM
w_r = weight for ridges
w_v = weight for valleys
h_{ex} = elevation of exaggerated DEM
h_{deep} = elevation of deepened DEM
h_{fl} = elevation of filtered DEM

As a result of these combinations, two elevation models are calculated, one appropriate for mountains and the other appropriate for lowlands. Because of the exaggeration, the ridges in the mountain DEM are higher than the surroundings, and the valleys in the lowland DEM are deeper. Ridges are clearly visible when shading is derived from the model for mountain areas (see Figure 7 left), and valley edges are accentuated on shaded relief derived from the model for lowland areas (Figure 7 right).

3.4 COMBINATION OF MOUNTAIN AND LOWLAND MODELS

The final step combines the mountain and lowland models into one elevation model. First an additional grid is created that indicates where the mountain and the lowland models are to be applied. Terrain parameters, such as slope or elevation range, can be used as indicators for this differentiation. In our approach, the slope values are used; these are extensively filtered, using a mean filter to remove small details and create compact areas of mountains and lowlands. An example of a filtered slope mask is shown in Figure 8: white indicates high values, and black indicates low values. The mountain and lowland models are combined according to equation (3), which is applied to each cell of the elevation models. The mountain grid is used in areas with high slope values, and the lowlands grid in areas with low slope values.

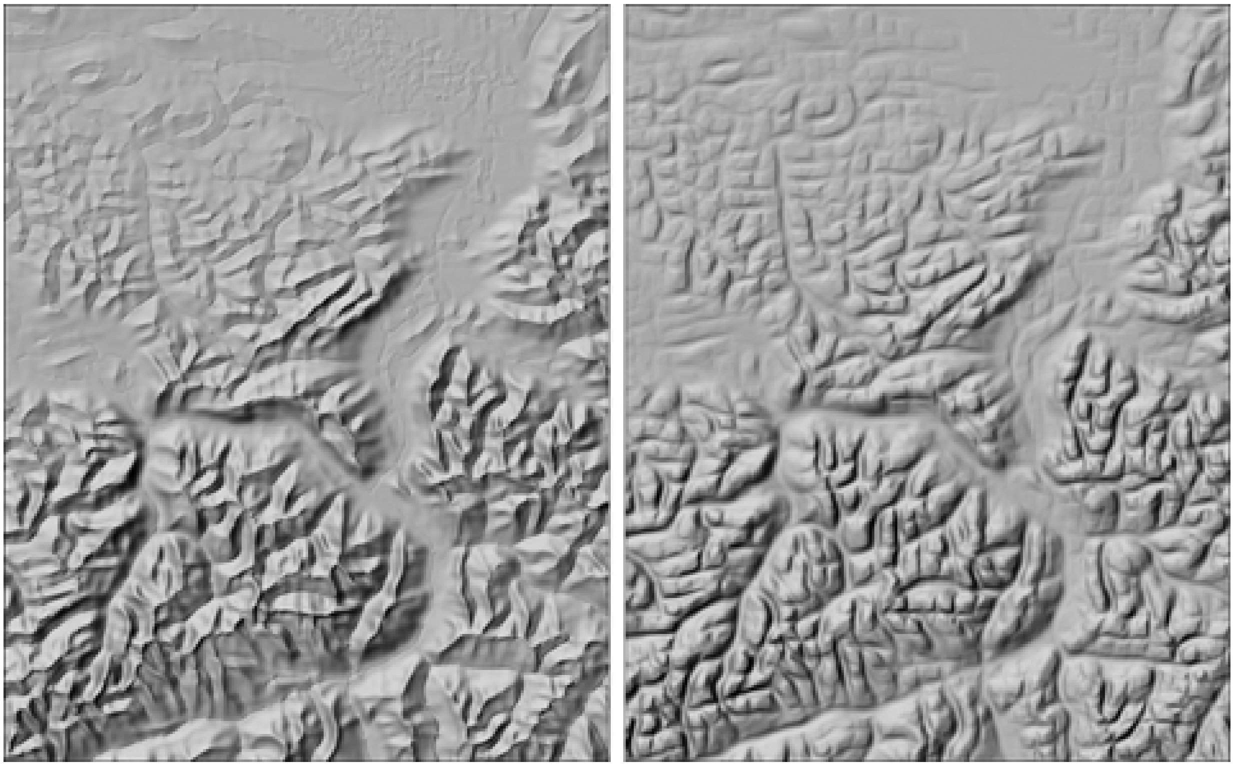

Figure 7. Relief shading appropriate for mountain areas (left) and lowland areas (right).

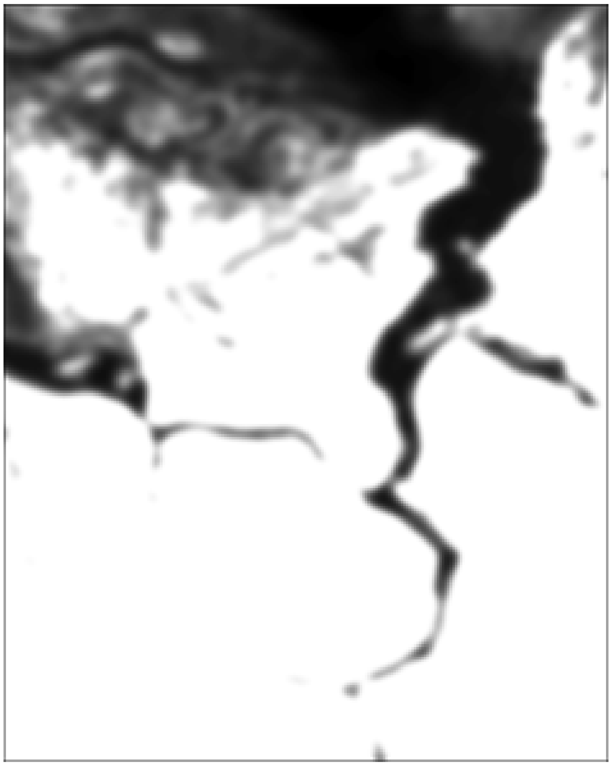

Figure 8. Slope mask for combining mountain and lowland areas

$$h_g = w_s \cdot h_{mount} + (1 - w_s) \cdot h_{low} \qquad (3)$$

where

h_g = elevation of generalized DEM
w_s = slope weight
h_{mount} = elevation of mountain DEM
h_{low} = elevation of lowland DEM

3.5 RELIEF SHADING

Figure 9 shows the shaded relief derived from the generalized elevation model. Shading was calculated with diffuse reflection, with the light source at an azimuth of 300° and an inclination of 45°; no local adjustment was made to the light direction. The calculated shaded relief was slightly corrected using raster graphics software (Adobe Photoshop CS2). First, small terrain irregularities appearing along mountain ridges were removed using a noise filter. To further improve the appearance, the grey values in flat areas were brightened; a uniform bright grey tone on flat areas is commonly used for manual shading. For this last operation, an additional greyscale image of slope values was used as a mask, and only the shading tone for the area with the lowest slope values was changed. Both Tom Patterson (1997) and Alex Tait (2002) describe details of this operation for Adobe Photoshop. We did not use any simulation of aerial perspective.

Figure 9. Shaded relief derived from generalized digital elevation model

4. Results

To evaluate the results of the method described in this paper, shaded relief derived from a generalized digital elevation model (Figure 9) was compared visually with the reference manually shaded relief (Figure 2). Because of the individual style of manual shading, exactly identical results cannot be expected. However, a comparison of the two shows that the level of detail achieved with our generalization procedure is fairly similar to the manually produced shaded relief. Many of the unnecessary small terrain details present in raw elevation data are removed, while the sharp structures of the main landforms are preserved. Ridges and valleys are clearly visible and are not disturbed by small terrain irregularities. The digitally shaded relief resembles the manual style in appearance, especially in mountainous areas.

Some improvements are necessary, especially in lowland areas, where too many small terrain structures are shown that are not present on the manual shaded relief. In addition, the main landforms of the lowland relief are not as clearly visible as they are in the manual shading; they should be more generalized and exaggerated. Differences between manual and automatic shading can also be observed in the high mountain areas, where the small relief details on the automatically shaded relief are not as sharp as in the manual equivalent. Furthermore, illuminated and shadowed terrain facets that meet crisply at the ridges depict high mountains on the manual reference. The digital sample in Figure 9 lacks this faceted appearance, and sinuous linear forms appear along the tops of a few ridges. The Photoshop noise filter successfully eliminates some of these linear forms; however, a few remain in the final image. Improvements are also necessary in the areas of lower mountains (e.g., the hilly landscape in the upper part of Figure 9), which, in comparison to the high Alps, look too bright. The shades in this part of the region should be intensified. Adjustments of local light direction and aerial perspective, which were not simulated in our approach, would likely further improve the appearance of the automatically shaded relief.

5. Conclusions

Digital elevation models provide detailed, worldwide terrain information and are commonly used for terrain analysis and visual exploration in GIS environment. Unfortunately, cartographers currently lack advanced methods for using these data to produce relief visualizations at a level of quality comparable to traditional, manually executed relief representations. Small-scale relief representations are especially difficult to automate, mainly because of the considerable generalization that is necessary.

The generalization procedure introduced here considers specific cartographic needs and allows for the creation of relief shading that is fairly comparable with the manual equivalent. The method can serve as an extension to the standard shading tools available in computer applications and can help cartographers to significantly improve their terrain visualization. The whole procedure can easily be performed with standard GIS and image-processing software. The shaded relief created with our generalization method should be further improved by introducing local adjustments of light direction and aerial perspective. In addition, other shading algorithms besides the diffuse reflection method need to be tested. Further research is necessary in order to apply the generalization method at other map scales (including greater scale reduction) and to test it in different regions with other relief types.

Development of novel automatic and semi-automatic methods that consider the requirements of cartographic visualization will certainly help to improve the quality of digital mapping and to adjust it to the standards developed by cartographers in the pre-digital era.

Author Information

Anna M. Leonowicz, Department of Geography, University of Zurich, 8057 Zurich, Switzerland. E-mail: anna.leonowicz@geo.uzh.ch. (Formerly at the Institute of Cartography, ETH Zurich.)

Bernhard Jenny, Institute of Cartography, ETH Zurich, 8093 Zurich, Switzerland. E-mail: jenny@karto.baug.ethz.ch.

Lorenz Hurni, Institute of Cartography, ETH Zurich, 8093 Zurich, Switzerland. E-mail: hurni@karto.baug.ethz.ch.

Acknowledgements

This research was supported by Swiss Federal Commission for Scholarships for Foreign Students and by ETH Zurich. The authors would like to thank the two anonymous reviewers for their valuable comments and suggestions.

Notes

1. There are no exact limits of what is defined as small, medium, and large scale. We follow the position of Robinson and others (1995) that maps at scales of 1:500,000 and smaller can be considered "small-scale."
2. Shuttle Radar Topography Mission (SRTM) home page, http://www2.jpl.nasa.gov/srtm/.
3. LandSerf, a free GIS software package for visualization and terrain analysis, is available at http://www.landserf.org/.

References

Bertolo, F. 2000. *Catchment Delineation and Characterisation: A Review* (Report No. JRC EU 19563 EN). Ispra, Italy: Joint Research Centre, European Commission. Available at http://agrienv.jrc.it/publications/pdfs/CatchRev.pdf

Böhm, R. 1997. *Kartographische Reliefdarstellung mittels digitaler Bildfilterverfahren* (PhD diss.). Vol. 15 of *Kartographische Bausteine*. Dresden: Institut für Kartographie, Technische Universität Dresden.

——. 2000. "Reliefgeneralisierung durch adaptive Bildfilterung von Rasterhöhenmodellen." *Kartographische Nachrichten* 6: 259–65.

Brassel, K. 1974. "A Model for Automatic Hill-Shading." *American Cartographer* 1/1: 15–27. doi:10.1559/152304074784107818

Burrough, P.A., and R.A. McDonnell. 2000. *Principles of Geographical Information Systems*. Oxford: Oxford University Press.

Deng, Y. 2007. "New Trends in Digital Terrain Analysis: Landform Definition, Representation, and Classification." *Progress in Physical Geography* 32: 405–19. doi:10.1177/0309133307081291

Gonzalez, R.C., and R.E. Woods. 2008. *Digital Image Processing*, 3rd ed. Upper Saddle River, NJ: Pearson.

Horn, B.K.P. 1982. "Hill Shading and the Reflectance Map." *Geo-Processing* 2: 65–144.

Imhof, E. 1982. *Cartographic Relief Presentation*. New York: Walter de Gruyter.

Jenny, B. 2001. "An Interactive Approach to Analytical Relief Shading." *Cartographica* 38/1–2: 67–75. doi:10.3138/F722-0825-3142-HW05

Keates, J.S. 1996. *Understanding Maps,* 2nd ed. Harlow, UK: Longman.

Kennelly, P.J. 2008. "Terrain Maps Displaying Hill-Shading with Curvature." *Geomorphology* 102: 567–77. doi:10.1016/j.geomorph.2008.05.046

Kennelly, P.J., and A.J. Stewart. 2006. "A Uniform Sky Illumination Model to Enhance Shading of Terrain and Urban Areas." *Cartography and Geographic Information Science* 33/1: 21–36. doi:10.1559/152304006777323118

Loisios, D., N. Tzelepis, and B. Nakos. 2007. "A Methodology for Creating Analytical Hill-Shading by Combining Different Lighting Directions." Proceedings of the International Cartographic Conference, Moscow [CD-ROM].

Mark, R. 1992. *Multidirectional Oblique-Weighted Shaded-Relief Image of the Island of Hawaii* (Technical Report No. OF92-422). US Geological Survey. Available at http://pubs.usgs.gov/of/1992/of92-422/

Minár, J., and I.S. Evans. 2008. "Elementary Forms for Land Surface Segmentation: the Theoretical Basis of Terrain Analysis and Geomorphological Mapping." *Geomorphology* 95: 236–59. doi:10.1016/j.geomorph.2007.06.003

Patterson, T. 1997. "A Desktop Approach to Shaded Relief Production." *Cartographic Perspectives* 28: 38–40. Available at http://www.shadedrelief.com/cp28/relief_tp.html.

——. 2001a. "Creating Swiss-Style Shaded Relief in Photoshop." Available at http://www.shadedrelief.com/shading/Swiss.html

——. 2001b. "Retro Relief Shading with Adobe Photoshop and a Wacom Tablet." Available at http://www.shadedrelief.com/retro/index.html

——. 2001c. "DEM Manipulation and 3-D Terrain Visualization: Techniques Used by the U.S. National Park Service." *Cartographica* 38/1–2: 89–101. doi:10.3138/8741-G618-5601-1125

——. 2001d. "Resolution Bumping GTOPO30 in Photoshop: How to Make High Mountains More Legible." Available at http://www.shadedrelief.com/bumping/bumping.html

Pennock, D.J., B.J. Zebarth, and E. de Jong. 1987. "Landform Classification and Soil Distribution in Hummocky Terrain, Saskatchewan, Canada." *Geoderma* 40: 297–315. doi:10.1016/0016-7061(87)90040-1

Prechtel, N. 2000. "Operational Analytical Hill Shading within an Advanced Image Processing System." In *High Mountain Cartography*, ed. M.F. Buchroithner, vol. 18 of *Kartographische Bausteine*. Dresden: Institut für Kartographie, Technische Universität Dresden. 85–98.

Rana, S. 2006. "Use of Plan Curvature Variations for Identification of Ridges and Channels on DEM." In *Progress in Spatial Data Handling: 12th Symposium on Spatial Data Handling*, ed. A.R. Riedl, W. Kainz, and G.A. Elmes. Berlin: Springer. 789–804. Available at http://www.springerlink.com/content/v405261t4516p646/. doi:10.1007/3-540-35589-8_49

Robinson, A.H., J.L. Morrison, P.C. Muehrcke, A.J. Kimerling, and S.C. Guptill. 1995. *Elements of Cartography*, 6th ed. New York: Wiley.

Schmidt, J., and R. Andrew. 2005. "Multi-scale Landform Characterization." *Area* 37: 341–50. doi:10.1111/j.1475-4762.2005.00638.x

Schmidt, J., I.S. Evans, and J. Brinkmann. 2003. "Comparison of Polynomial Models for Land Surface Curvature Calculation." *International Journal of Geographical Information Science* 17: 797–814. doi:10.1080/13658810310001596058

Shary, P.A., L.S. Sharaya, and A.V. Mitusov. 2002. "Fundamental Quantitative Methods of Land Surface Analysis." *Geoderma* 107: 1–32. doi:10.1016/S0016-7061(01)00136-7

Tait, A. 2002. "Photoshop 6 Tutorial: How to Create Basic Colored Shaded Relief." *Cartographic Perspectives* 42: 12–17.

Tribe, A. 1992. "Automated Recognition of Valley Lines and Drainage Networks from Grid Digital Elevation Models: A Review and a New Method." *Journal of Hydrology* 139: 263–93. doi:10.1016/0022-1694(92)90206-B

Weibel, R. 1992. "Models and Experiments for Adaptive Computer-Assisted Terrain Generalization." *Cartography and Geographic Information Systems* 19: 133–53. doi:10.1559/152304092783762317

Wilson, J.P., and J.C. Gallant, eds. 2000. *Terrain Analysis: Principles and Applications*. New York: Wiley.

Wood, J.D. 1996. *The Geomorphological Characterisation of Digital Elevation Models*. PhD diss., University of Leicester, UK. Available at http://www.soi.city.ac.uk/~jwo/phd

Yoeli, P. 1959. "Relief Shading." *Surveying and Mapping* 19: 229–32.

——. 1965. "Analytical Hill Shading." *Surveying and Mapping* 25: 573–79.

——. 1967. "Die Richtung des Lichtes bei analytischer Schattierung." *Kartographische Nachrichten* 2: 37–44.