# Visualizing the Planimetric Accuracy of Historical Maps with MapAnalyst

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## Abstract

MapAnalyst is a new software application for the visualization and study of the planimetric accuracy of old maps. It illustrates local map distortion by generating distortion grids, displacement vectors, and new isolines of scale and rotation. MapAnalyst additionally computes the old map's scale and rotation, as well as statistical indicators summarizing the map's geometric accuracy. It offers a user-friendly interface and is freely available for all major computer platforms at http://mapanalyst.cartography.ch/. Map historians are invited to use MapAnalyst, and are encouraged to consult and improve the free Java source-code.

This article describes the steps leading to visualizations of a map's planimetric accuracy. It provides basic algorithmic information that is necessary for the understanding and correct interpretation of displacement vectors and distortion grids. It also introduces isolines of equal scale and rotation, a new type of accuracy visualization. The last section interprets sample visualizations for an eighteenth century map.

### Visualizing Accuracy

The planimetric accuracy of a historical map is the extent to which distances and bearings between identifiable objects on a map coincide with their true value (Blakemore and Harley 1980).[1] Visualizations of planimetric accuracy illustrate how rotation, shearing, shrinkage, and stretching vary locally on the map.

The applications of accuracy analyses are twofold. An analysis may support or refute a hypothesis about technical aspects of map creation. The map historian may, for example, verify assumptions about the surveying methods and source maps used to compile the map or examine its underlying projection and geodetic reference. Alternatively, an analysis may indicate the geometric reliability of information extracted for historical research (e.g., studies of changes in land use, vegetation, or coastal erosion; Blakemore and Harley 1980). Graphic presentations of the planimetric accuracy allow for discovering and understanding new facts about a map and are an excellent means to illustrate these findings. It is thus essential to have available a range of complementary visualizations that illustrate the local geometric characteristics of the map and emphasize punctiform or spatially limited outliers.

Various computer-aided techniques exist for the visualization of planimetric distortion and accuracy. Gustav Forstner and Markus Oehrli (1998), as well as Evangelos Livieratos (2006), provide concise overviews of techniques developed for the graphical analysis of old maps. Most visualizations derive from two sets of corresponding points; that is, one set originates from a modern reference map and is considered to be perfectly accurate, while the points of the other set is drawn from the old map and is supposed to be inaccurate. Before a visualization can be generated, the two sets of points must share a common coordinate system. Affine transformations bring the two coordinate systems into coincidence by converting one set of points to the coordinate system of the other set. This is achieved by scaling, shifting, and rotating one set in such a way that the differences between the two sets are minimal according to the least-squares method. Compared to other types of transformations, affine transformations offer the advantage of

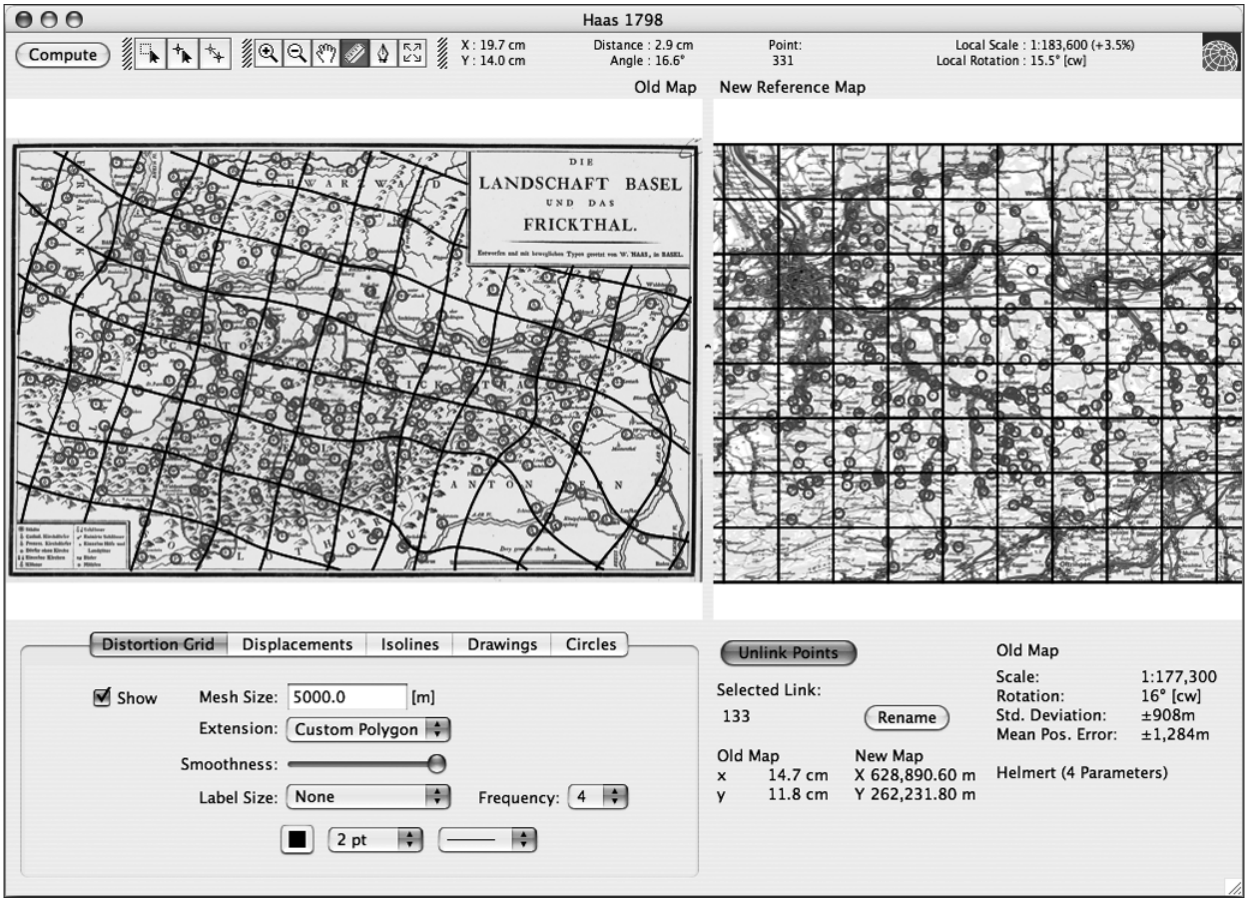


**Figure 1.** Screenshot of MapAnalyst's main window (version 1.2.1).

providing estimations of the scale and the rotation of the old map.[2]

An analysis with MapAnalyst begins by importing images of an old map and a georeferenced modern map (see Figure 1). The user then identifies corresponding locations on the two maps and interactively places pairs of points at these locations. Finally, he or she chooses the appropriate type of affine transformation and selects the parameters for the desired accuracy visualization. The computation of the various visualizations is quick, which facilitates an iterative approach. For example, changing the mesh size of a distortion grid or computing the scale of a sub-area of the map is a matter of seconds. MapAnalyst can then export the visualizations it has generated to various graphics formats.

### DISPLACEMENT VECTORS

Displacement vectors are algorithmically the simplest visualization technique and are easy to understand (see Figure 2a). Each vector line starts at a point previously identified in the old map and ends at the position where the point would be if the old map were as accurate as the modern reference map. This endpoint results from an affine transformation between the two sets of points. Exceedingly long vectors are easily identifiable; these indicate outliers that are due to gross positional errors in the map.

### DISTORTION GRIDS

The rotated, compressed, or enlarged meshes of a distortion grid reflect the local deformation and rotation of the old map (see Figure 2b).[3] When manually constructing a distortion grid, the map historian fits grid lines into a field of reference points based on rather subjective estimations.[4] Computer-based construction objectifies and accelerates this rather tedious manual construction process.

To construct distortion grids, MapAnalyst uses a method developed by Dieter Beineke (2001), which is based on the multi-quadratic interpolation introduced by R.L. Hardy (1971). This method offers the advantage of minimizing the influence of points with gross errors and prevents the generation of closed circular lines. In short, the distortion grid is constructed as follows. First, displacement vectors on the old map are computed, as described above. The displacement vectors are then used to determine the parameters of the multi-quadratic interpolation. Thus the multi-quadratic interpolation provides

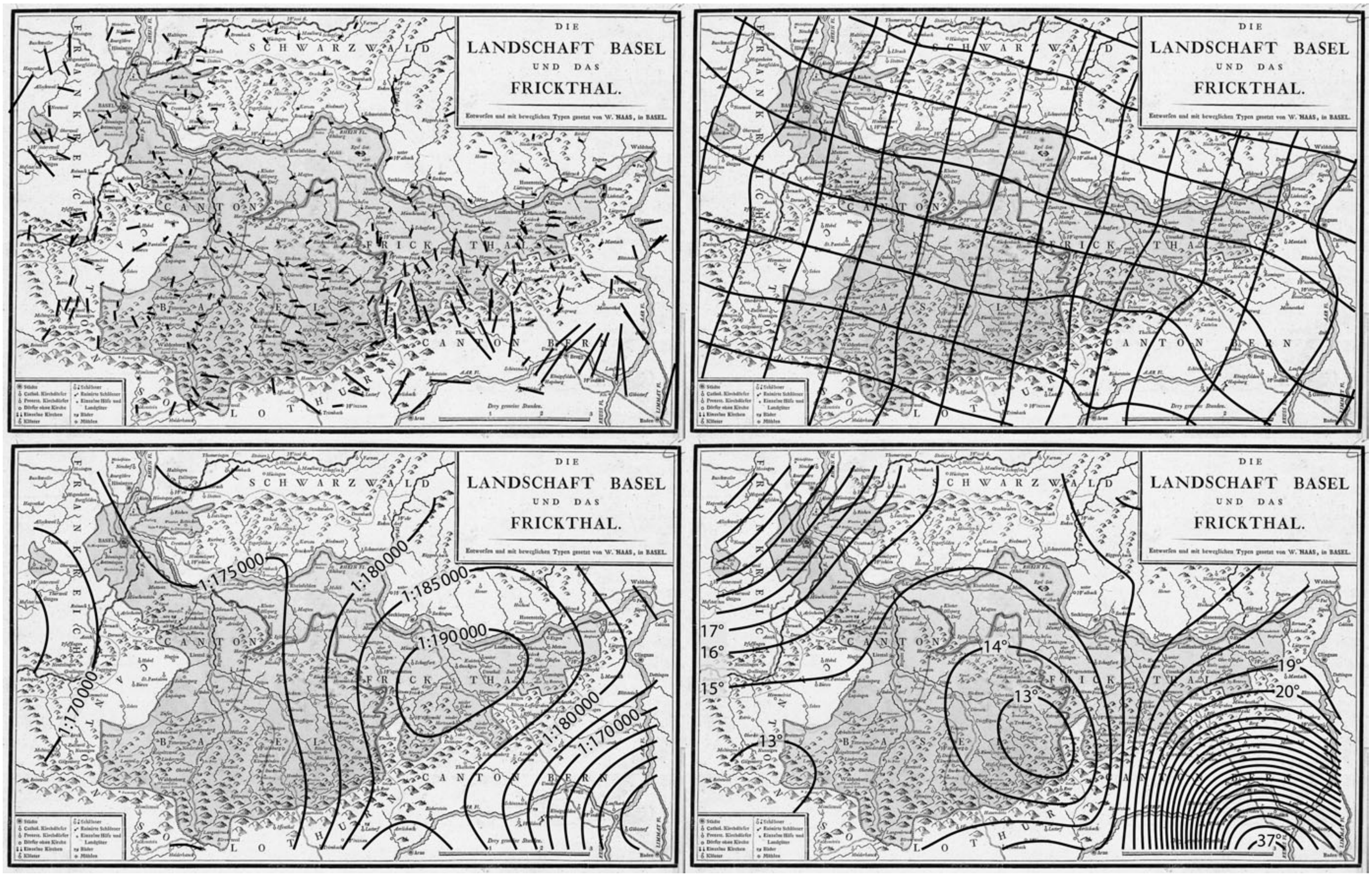


**Figure 2.** W. Haas, *Die Landschaft Basel und das Frickthal* (1798). Approximately 21 × 34 cm; scale 1:177,300; rotation 16°. Overlaid by (clockwise from bottom left): (a) displacement vectors, (b) a distortion grid, (c) scale isolines, and (d) rotation isolines.

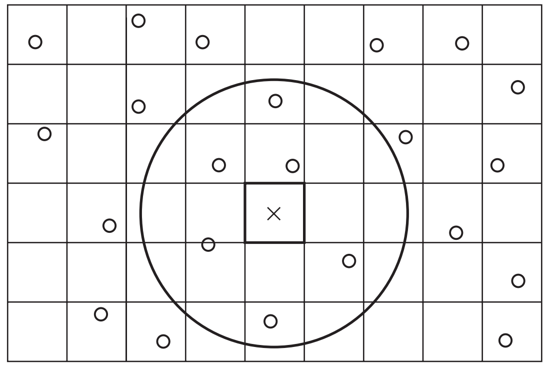

**Figure 3.** In this example, six points inside the circle determine the scale and rotation for the central cell.

estimated displacement vectors for arbitrary points on the study map. The penultimate step involves constructing a regular grid in the coordinate system of the reference map and its affine transformation, which results in a regular, but scaled and rotated, grid in the coordinate system of the old map. In the final processing step, this grid is distorted using the multi-quadratic interpolation initialized earlier.

It is important to note that other authors have used alternative computer-assisted methods, which produce distortion grids that may differ in shape (Tobler 1966, 1994; Weis 1985).[5]

#### ISOLINES OF SCALE AND ROTATION

Isolines are a new means of visualizing local variations of scale and rotation. They connect locations of equal scale and rotation (see Figures 2c and 2d). The underlying algorithm uses two invisible raster grids that hold regularly spaced scale and rotation values. It proceeds in three steps: first, it creates two raster grids that will hold scale and rotation values; then it computes a scale and a rotation value for each cell of the raster grids; finally, the algorithm extracts isolines from the raster grids using a contouring algorithm.

Raster grids and derived contour lines are commonly used for modelling and visualizing spatially continuous phenomena (e.g., digital elevation models). The novel part of the proposed method is the estimation of local scale and rotation values. When computing the values for a particular cell instead of using all available pairs of points, we use only those points that lie within a circle around the cell (Figure 3). We scale and rotate these points from one map onto the corresponding points of the other map, again using an affine transformation and the least-squares method. As explained above, the affine transformation provides estimations of the scale factor and the rotation angle, which are stored in the raster grids. It is intuitive that points more distant from the centre of the circle should have a smaller influence on the computation than points closer to the centre. This is achieved by individually weighting each point, depending on its distance from the centre of the circle, using a Gaussian bell curve.

The shapes of the resulting isolines depend to a great extent on the radius of the circle. With a small radius, isolines will reflect local variations, while a large radius of influence will generate smoother isolines. A careful choice of the radius is therefore a crucial step when applying this method.

### Analysis of an Eighteenth-Century Map

The 1798 map ‘Die Landschaft Basel und das Frickthal’ by W. Haas stands out for its use of movable type for text placement (Harris 1975). It depicts a part of today’s Switzerland with neighbouring Germany and France. At the time the map was produced, the eastern portion belonged to Austria-Hungary. We produced a series of accuracy visualizations to illustrate a typical application of MapAnalyst and to test the hypothesis that this map is a combination of at least two different base maps. Indeed, there is no historical evidence of a contemporary survey covering the entire area. Scale, rotation, statistical indicators, and visualizations were computed with 343 pairs of corresponding points (Figure 2). We used a four-parameter Helmert transformation to match the coordinate systems. The Helmert transformation (also called ‘‘conformal’’ or ‘‘similarity’’ transformation) shifts the map in two directions, applies a single scale factor, and uses a single rotation angle. It does not distort the shape or geometry of the map (Livieratos 2006).

The displacement vectors show locally consistent patterns, except for large outstanding vectors in the south-eastern part of the map. Vectors in the eastern part are generally south or east oriented, whereas vectors in the western part are roughly west oriented. The distortion grid, with a mesh size of 5000 × 5000 m, clearly visualizes the general rotation of about 16 degrees, which reflects the declination of the contemporary magnetic field.[6] The meshes are fairly homogeneous in the western part of the map, whereas local distortions are visible in the eastern part. The scale and rotation isolines were computed with a circle radius of 20,000 m. The scale isolines reveal that the map depicts the western section at approximately 1:175,000, while the eastern part is shown at a considerably smaller scale of approximately 1:190,000. The rotation isolines indicate important distortions in the south-eastern and north-western corners of the map.

A summary of the four visualizations makes it seem highly probable that this map is not based on a single geodetic survey covering the complete area. Instead, it was probably compiled from at least two base maps, as the patterns of the displacement vectors and the clearly diverse scale of the eastern and the western part indicate. The base map for the western part was possibly geometrically more accurate than the one for the eastern part. The south-eastern portion of the map serves only for general geographic reference, since information density diminishes and geometric distortion increases here, as indicated by all four visualizations shown in Figure 2. Another question to investigate for this particular map is the influence of movable type placement on planimetric accuracy, which is not further discussed here.

## Conclusion

MapAnalyst offers map historians interesting options for the analysis and visualization of the planimetric properties of historical maps. Compared to manual techniques, MapAnalyst greatly simplifies and accelerates the analysis and increases the reliability of the results. The user-friendly interface facilitates the creation of a variety of complementary visualizations within very short time. For more information about MapAnalyst, please visit the Web site <http://mapanalyst.cartography.ch/>, which provides the software for downloading as well as sample data, tutorials, and additional documentation.

## Acknowledgements

The authors would like to thank the following individuals and institutions: Martin Rickenbacher and swisstopo, Bern (for providing the control points and the scan for Figure 2); Jana Niederöst, Dante Salvini, and Helen Jenny, ETH Zurich; Dieter Beineke, Universität der Bundeswehr, Munich; Matthias Kistler, swisstopo, Bern; Markus Oehrli, Zentralbibliothek Zurich; Nick Millea, Bodleian Library, Oxford; Waldo R. Tobler, University of California, Santa Barbara; and Ingrid Kretschmer, University of Vienna.

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## Notes

1. Michael Blakemore and J.B. Harley (1980) differentiate between geodetic, planimetric, and topographic accuracy. Geodetic accuracy is the quantification of errors in the astronomically based graticule. Topographic accuracy measures the quantity and quality of the information about landscape objects in a map.
2. Determining the scale by an affine transformation is preferable to averaging the scale from a series of distances measured in the old and new reference maps, since a transformation is more objective and less error-prone.
3. Distortion grids were first used by Hermann Wagner in 1895 (Wagner 1895). In more recent literature, the invention of distortion grids is sometimes incorrectly attributed to Eduard Imhof, as explained by Forstner and Oehrli (1998).
4. Forstner (1998) presents two manual techniques for the construction of distortion grids.
5. Waldo Tobler (1966, 1994) uses bi-dimensional regression. Ingrid Weis (1980) digitally simulates manual construction in one method and uses distance weighting in another. Various authors derive distortion grids from triangulated networks.
6. The Geomag model of the historical geomagnetic field by the British Geological Survey, included in the software GeoMag, interpolates a magnetic declination of $-17.9^\circ$ for the study map. Hence, the difference between the orientation of the map and the historical magnetic declination is $2^\circ$ (after correction for the convergence of the meridian of the reference map). For the GeoMag software by Garry Petrie, see www.resurgentsoftware.com/geomag.html.

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