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Swiss-Style Colour Relief Shading Modulated by Elevation and by Exposure to Illumination

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Maps with coloured relief shading, modulated by elevation and by exposure to illumination, present topography in a particularly vivid and descriptive manner. Carefully modulated illumination and shading shown in continuous colour tones simulate the third dimension of topography, helping map-readers to conceive more easily the terrain's important landforms. Since the end of the 19th century, cartographers have developed a wide variety of colour schemes. The first part of this paper illustrates the graphical and technical developments of the past, leading to Swiss-style coloured relief shading; the second part presents a new computer-based method for colourizing grey-shaded relief inspired by classic colour schemes. The method uses a colour look-up table and a digital elevation model. The colour look-up table has colours for all combinations of initial grey values and terrain elevations. It is constructed from interactively placed colour reference points.

INTRODUCTION

Colour is an excellent means of enhancing the simulated three-dimensionality of a shaded relief, and offers the reader necessary graphical clues to comprehend complex topography more easily. One can identify two related approaches:

1. hypsometric tinting
2. colours modulated by elevation and by exposure to illumination.

Hypsometric tinting may be regarded as the simpler technique, assigning colours solely by elevation. Hypsometric colours are most common in small-scale topographic maps, either applied as continuous gradients or as discrete layers, often in combination with a shaded relief. Austrian cartographers were among the first to base hypsometric colour sequences on theoretical considerations (Kretschmer, 2000). For example, Franz von Hauslaub propagated the principle 'higher is darker' (Hauslaub, 1864), or Karl Peucker suggested increasing the saturation of the colours with rising elevation (Peucker, 1898). His colour scale ends in red, and still influences contemporary European small-scale mapping. In contrast to these rather colourful schemes, Imhof (1982; Figure 1) proposes relatively light and more natural colours, with the colour ramp following the principle 'higher is brighter'.

The three-dimensional appearance of a relief representation can be improved by additionally modulating colours by exposure to a source of illumination. Hence, colours vary with increasing elevation and with the variation towards a virtual light source – with the main purpose of emphasising the terrain's characteristic shapes. As with hypsometric colours, cartographers have invented a large variety of exposure-modulated colour and greyscale schemes. A typical example covers low-lying flat planes with relatively cold greenish or bluish colours to simulate the effect of aerial perspective, which is due to haze and other particles in the atmosphere. Aerial perspective is increasing in density with the distance from the observer, and is used as a graphical device to differentiate between high mountain summits and lower, more distant lowlands. On sun-lit slopes colours change with elevation from blue-green, to green or olive, then yellow or olive-yellow and end with white. On shadowed slopes the sequence changes from green-purple to olive-purple and blue-purple and ends with dark purple.

Various elements of cartographic relief depiction have been successfully transferred to the digital realm in recent years, e.g. grey shading (Jenny, 2001), cliff drawing (Hurni *et al.*, 2001), and spot height placement (Palomar and Pardo, 2004). Coloured relief shading modulated by illumination, however, has received little attention so far. Apart from a technique based on raster graphics software described by Tait (2002) and Patterson (date not known), no specialized methods have been developed. In short, there is, as yet, no quick and easy method for use by non-experts in raster graphics software.

The first part of this paper provides an overview of the historic development of colour-modulated relief shading. It

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Figure 1. Eduard Imhof, 1895–1986
Figure 2. Fridolin Becker, 1854–1922

starts with pioneering work at the end of the 19^{th} century, when the Swiss Alpine Club published the first coloured relief maps printed with chromolithography. These early maps initiated a period of experimentation, leading to a variety of new colour schemes. Chromolithographic printing of coloured shading was subsequently replaced by a photomechanical process that considerably simplified production by deriving the printing plates from a single grey shading.

The second part presents a new computer-based method that colours shaded relief by modulating colour with illumination and elevation. The software described applies colour on shaded relief produced by computer-based analytical or traditional manual shading. The algorithm uses a colour look-up table and a digital elevation model. The colour look-up table holds interpolated colours for all combinations of initial grey values and terrain elevations. We describe an interactive method to construct the colour look-up table from colour reference points that the user places on the grey shading.

HISTORIC DEVELOPMENT

In the past, cartographers have developed a wide variety of colour schemes for depicting terrain. A close look at the historic development of coloured relief shading is of value, since it can reveal instructive details and provide modern day cartographers with valuable inspiration. For this purpose, we have restricted this historic overview in two ways. First, we deliberately concentrate on developments introduced by Swiss cartographers. This somewhat artificial restriction ignores the long-established international exchange of ideas and knowledge within the cartographic community, which led to alternative solutions in other parts of the world. But, Swiss cartographers did have a leading role in the development of coloured relief shading and therefore merit mention here (Cavelti Hammer *et al.*, 1997). In particular, they managed to combine contour lines, shading, rock and scree drawing and introduced the use of naturalistic colour schemes to create the so-called Swiss-style map graphics (Cavelti Hammer *et al.*, 1997). Also, we limit this historical review to printed maps, ignoring early manuscript maps with coloured relief shading, such as Hans Conrad Gyger's outstanding map of the Canton of Zurich of 1664 (Cavelti Hammer *et al.*, 1997).

Figure 3. Canton of Glarus, the first printed Swiss-style map deliberately simulating the effect of aerial perspective

Printing technology played a determining role in the development of coloured relief shading. Cartographers were constantly searching for and adapting technical improvements in colour reproduction. The first technology that allowed mass production of coloured relief maps was chromolithography, a multi-colour lithographic printing process, which reached its technical maturity at the end of the 19^{th} century. Lithography uses stones as printing plates with a smooth surface – rather than a raised (woodcut or movable type) or incised surface (engraved plate). The transfer of printing ink is accomplished by chemical, rather than physical action, based on the antipathy of water and grease (Ristow, 1975). The combination of differently coloured plates produces the desired colour gradients.

The Swiss Alpine Club introduced the first chromolithographic maps with modern coloured relief shading and contour lines. In 1863/64, the club started adding a map of a section of the Alps to its annual club book. Rudolf Leuzinger, a mountain cartographer, who also gained fame for his cliff drawings, created these first relief maps. In 1865, the club published Leuzinger's first coloured map of the Lukmanier–La Greina alpine area. This map at a scale of 1:50 000 with its discreet brown-grey colours, is a first tentative experiment, which does not yet show the terrain in the colourful and intuitive representation of later maps.

Fridolin Becker, professor at ETH Zurich (Figure 2), further improved Leuzinger's ideas by searching for more natural colours (Schertenleib, 1997). In 1889, the Swiss Alpine Club published his map of the Canton of Glarus (Figure 3). It depicts sun-lit slopes in yellow-green, and shadowed slopes in dark green. Becker was the first to

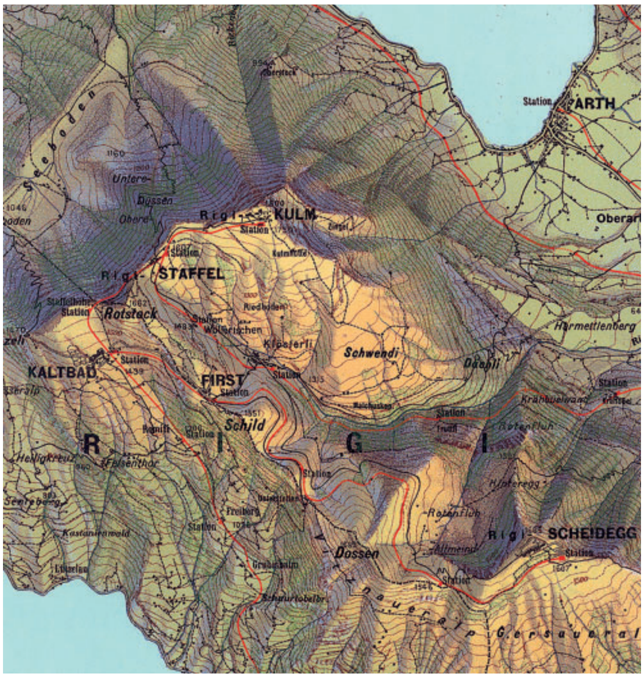

Figure 4. A strong colour contrast between yellow sun-lit slopes and purple shadow slopes enhances the three-dimensional effect. The relief shading uses unusual southern illumination

simulate the effect of aerial perspective in printed maps by gradually sharpening colour contrast towards the highest peaks. The colours of Becker's map caused a sensation among experts and tourists – the public admired the three-dimensional effect produced by the coloured shading. Becker, although not entirely satisfied with his map, clearly achieved his goal: a brief look at the map is enough to grasp the shape of the terrain. After this outstanding map, Becker continued to experiment with alternative colour and illumination schemes (e.g., the map of Mount Rigi with southern illumination in Figure 4), and produced a series of topographic maps at various scales.

Leuzinger, Becker and other cartographers first created an original map with watercolours, which the lithographer transferred to colour-separated lithographic plates. One could compare the lithographer's work to the functioning of a modern scanner that converts a multicoloured original to colour-separated channels. Excellent co-ordination between the cartographer and lithographer was essential because of the difficulty of predicting the interaction of multiple colours.

At the beginning of the 20th century, the so-called Siegfried maps became available to the public – large-scale maps made by the Swiss Office of Topography with detailed contour lines and rock drawing, but without shaded relief. The Swiss Alpine Club reacted by ceasing further map production, which put an end to its role as leading initiator in coloured relief mapping. It was Eduard Imhof (Figure 1), Becker's successor as professor at ETH Zurich, who subsequently developed improved techniques. He used school maps and atlases as an experimental field. One of Imhof's most important contributions was the development of a photomechanical process for the production of coloured shaded relief (around 1945). This procedure derives a series of printing plates from a single monochrome shading by photomechanical reproduction. Figure 5 shows an example of a set of six different colour plates that were produced from a single greyscale shading. The first light green plate is a rather soft copy of the original shaded relief. The second plate contains five hypsometric grades. The yellow tint of plate three simulates sun-lit slopes. It is a photographic negative of the original shading that applies saturated yellow to areas oriented towards the illumination source. Flat areas are masked to avoid yellow colouration in even lowlands. Plates 4 to 6 are copies of the original shading with increasingly higher contrast. The last two plates simulate aerial perspective by strengthening shades and contrast at the highest peaks. Figure 6 shows a section of the school wall map of the Canton of Grisons, one of Imhof's masterpieces produced by photomechanical reproduction.

Coloured relief mapping modulated by illumination was originally developed for large-scale maps of mountainous areas, but has been successfully adapted to flatter topography and to maps at medium and small scale. Figure 7, for example, shows a colour scheme developed by Imhof for small-scale maps that is characteristic for the former Swiss High School Atlas (Schweizerischer Mittelschulatlas) (Imhof, 1964). The colour scale 'ranges with increasing elevation from a light grey-blue-green by light olive to brownish-reddish yellow tones, and finally to white' (Imhof, 1961).

Photomechanical reproduction considerably simplified the preparation of printing plates. Thenceforward, a cartographer could produce grey shadings instead of colour manuscript originals, and thereby simplify the workflow. Still, photomechanical reproduction was a laborious and expensive process. For example, Bantel (1973) mentions that the creation of five printing plates for the shaded relief of Figure 8 required a total of 55 intermediate masks and films.

During the second half of the 20th century, important progress was made in the area of reproduction techniques, which cartographers adapted gratefully for their purposes. Colour separation using conventional large-scale reproduction cameras allowed for improved offset colour printing. Imhof took advantage of these advances for the more artistic map image 'Relief de la Suisse', his last important work that was painted with watercolour and printed with three colour plates (Figure 9).

With the advent of affordable personal computers and digital elevation models, cartographers started adopting digital techniques for coloured relief presentation. Generating hypsometric tints is straightforward from an algorithmic as well as from a user's point of view. Today, many software packages are available for applying colour ramps on digital elevation models, allowing the cartographer to specify an arbitrary number of colour classes (see Imhof, 1982, for a discussion of colour gradients).

Yellow toning is an alternative method for improving the three-dimensional appearance of a terrain that leads to discreet graphic representations. The grey relief shading is combined with a bright yellow tone on the sunlit slopes. The yellow highlights are derived from the grey shading by inverting its grey values (Patterson, date not known). Flat

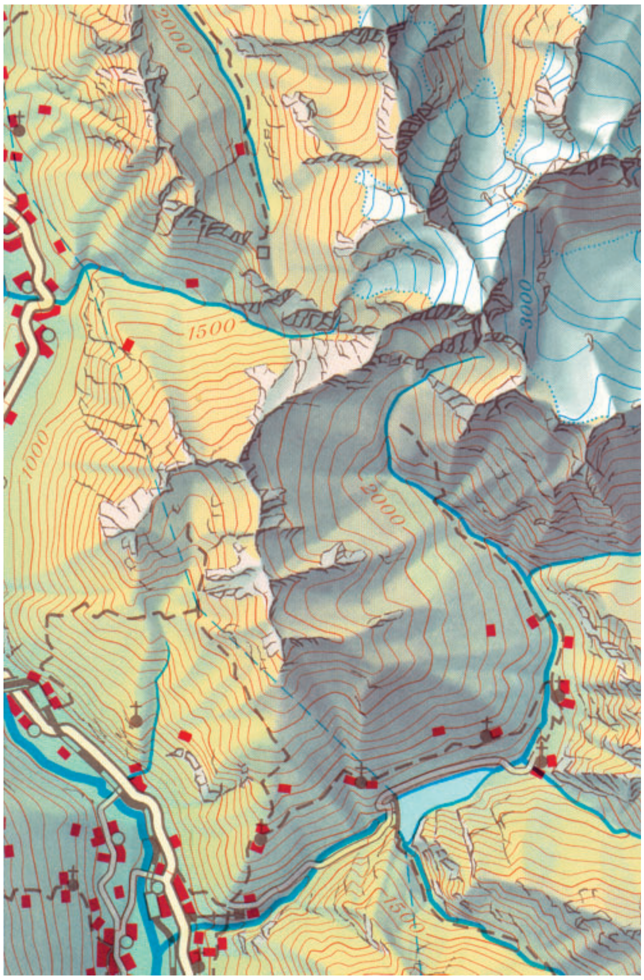

Figure 6. Colours photomechanically derived from a grey relief shading for the Grisons school map

areas should be masked to avoid yellow toning. The effect of yellow highlights can be enhanced by a combination of hypsometric tints with shaded relief and coloured shades (Tait, 2002). This procedure is a simulation of Imhof's analogue photomechanical technique using raster graphics software such as Adobe Photoshop.

A complementary group of techniques aims at visualising the slope and aspect of the terrain (or orientation towards a light source). Moellering and Kimerling (1990) visualise aspect classes using varying colour hues. Brewer (1993) improves the technique by additionally varying colour saturation by the terrain's slope, and by adjusting lightness to produce a systematic perceptual progression. Kennelly and Kimerling (2004) aim at accentuating terrain details that are not visible in simple computer-derived hill shading by varying colour with aspect direction. Hobbs (1995, 1999) strives for a similar goal: he tries to clarify the depiction of landforms by combining three shadings produced by three different directions of illumination. The three shadings are assigned to the red, green and blue channel of an RGB image. The four techniques mentioned

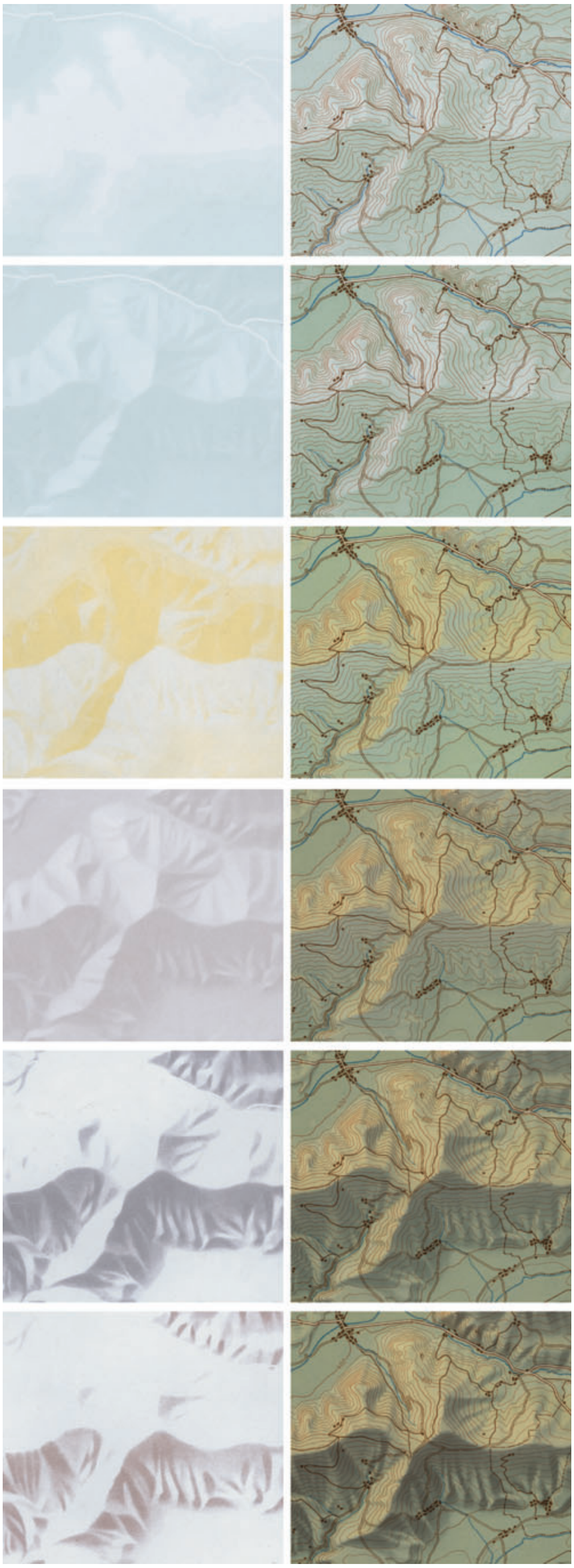

Figure 5. Left: Six coloured printing plates for relief depiction. Right: Successive combination of the plates with additional information, printed with another six printing plates

Figure 7. A section of a typical example from the Swiss Mittelschulatlas with colour modulation adapted to small scales

undeniably have valuable applications, but the colourfulness of the resulting images generally impedes their use as background information for topographic maps. Nevertheless, the techniques merit attention, since they are related to our approach by their goal of accentuating important landforms by varying colour with the terrain's orientation.

A NEW METHOD FOR DIGITAL SWISS-STYLE SHADING

While Swiss-style relief depiction has been brought to an excellent level during a long process of improvements, the digital era offers new tools that can be used to simulate established techniques with computer software (see for example Tait 2002 as mentioned above). While this

Figure 8. Coloured relief for a school map produced by photomechanical production, printing with five plates

Figure 9. Original map painted with watercolour, reproduction by four-colour printing

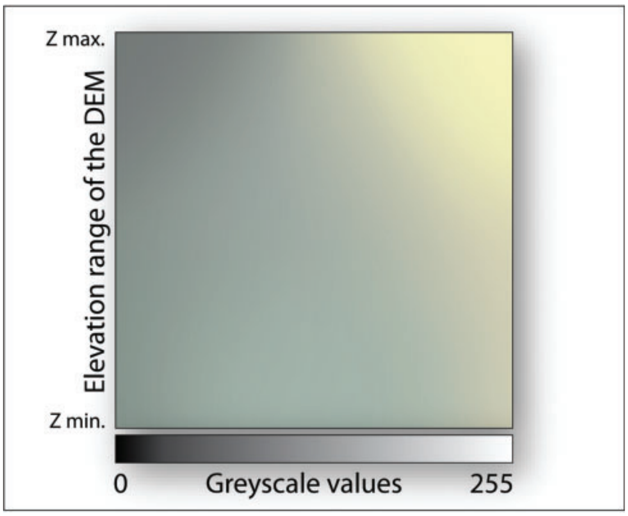

Figure 10. Colour look-up table. Grey values and elevations both discretized in 256 steps

approach yields good results, we are certain that modern information technology offers many exciting new ways for cartographic relief presentation that remain to be discovered. New digital techniques should offer tools that are easy to use by the novice and inexperienced user, and should bring well-proven concepts to the digital realm. It is in this spirit that we developed the method described below.

Our new digital method transfers classic colour schemes to the digital realm by modulating colour with elevation and exposure to illumination. We implemented an easy-to-use program that allows for constructing and comparing alternative colour schemes, and offers fine-tuning of the colours by the interactive placement of coloured reference points. The required input consists of a digital elevation model (DEM) and a greyscale shading covering the same area. The method allows for colouring shadings that were drawn manually or computed from digital elevation models. The algorithm uses a two-dimensional look-up table containing colour definitions (Figure 10). On the table's vertical axis, colour varies with elevation between the lowest and the highest point of the DEM. On the horizontal axis, colour changes with the grey value of the original shading between black (left) and white (right). Hence, the table defines one colour for each combination of a specific elevation and greyscale value.

When generating the new coloured relief image, the software determines for each pixel the corresponding elevation from the DEM and the greyscale value from the original shading. These two values are then used to extract the corresponding colour from the look-up table. This colour is finally assigned to the pixel in the new shading (Figure 11).

The basic idea of using a look-up table is related to the concept of the so-called reflectance map for greyscale shading proposed by Horn (Horn, 1980). Horn extracts slope values in west-to-east and south-to-north direction from the digital elevation model, and uses these values to find the greyscale values from a look-up table.

A relief's colour scheme needs to be adapted to the individual characteristics of a map and to the properties of the depicted terrain. Since in our method, a look-up table determines the colour scheme, a software application must offer user-friendly interactive tools to construct customized look-up tables. We explored and compared two alternative ways of specifying look-up tables, using a prototype application developed with the C++ and Objective-C programming languages on Mac OS X.

In the first approach, the user selects a freely definable colour from a standard colour picker, and places this colour in an interactive control panel displaying the look-up table

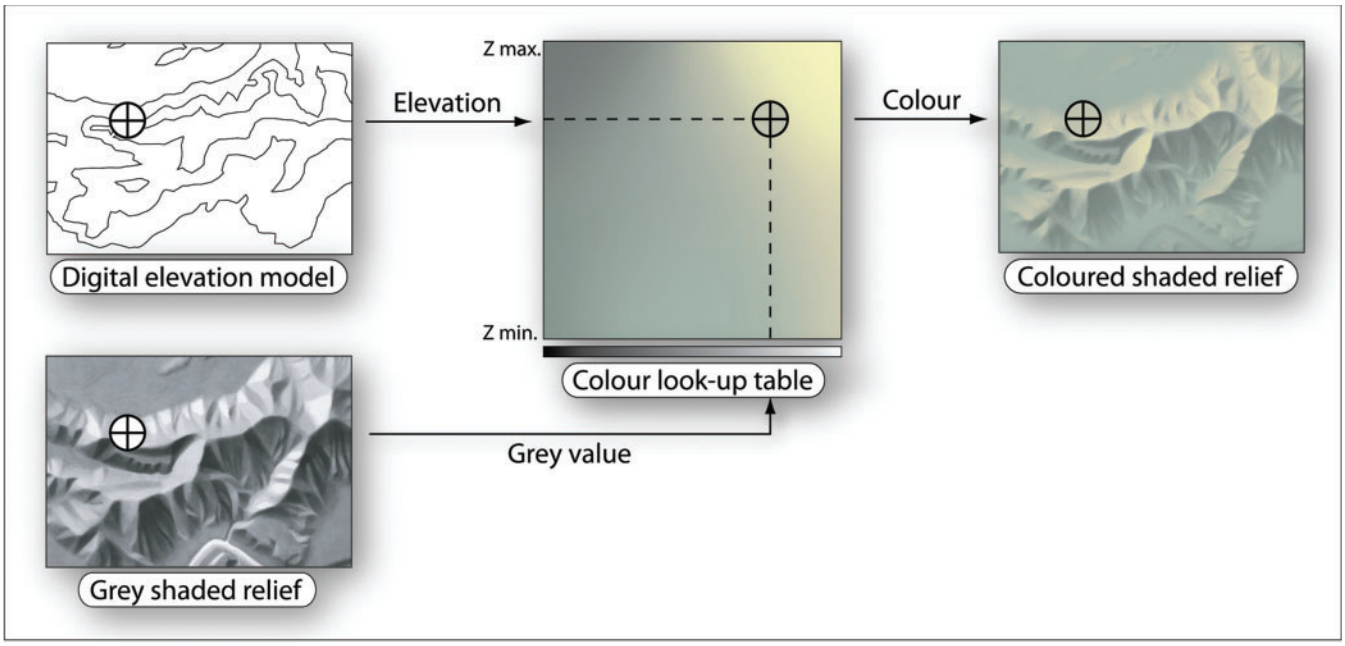

Figure 11. Tinting a shaded relief. A colour is extracted from the look-up table based on the original grey value and the elevation extracted from a DEM

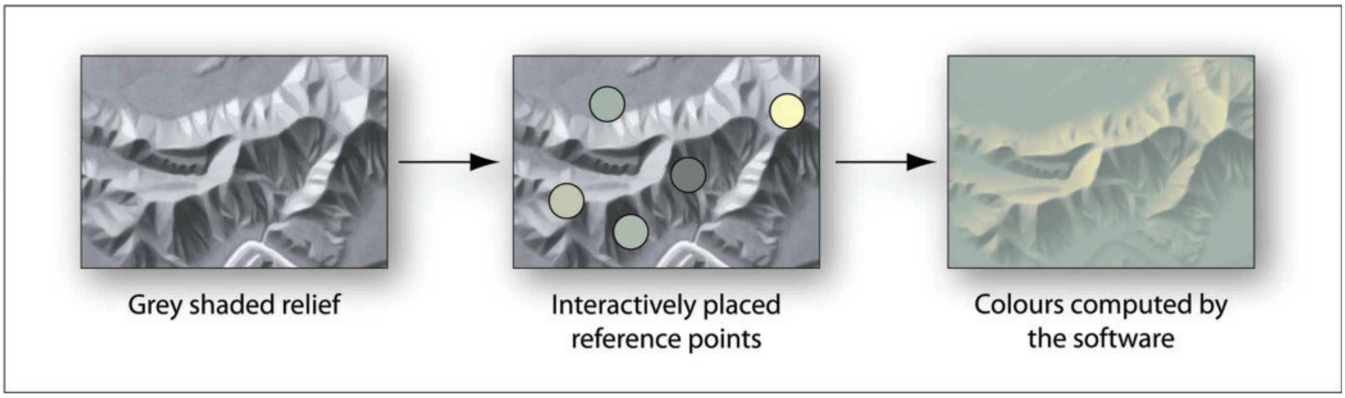

Figure 12. The workflow from a user's perspective. The user places coloured reference points on the grey shading

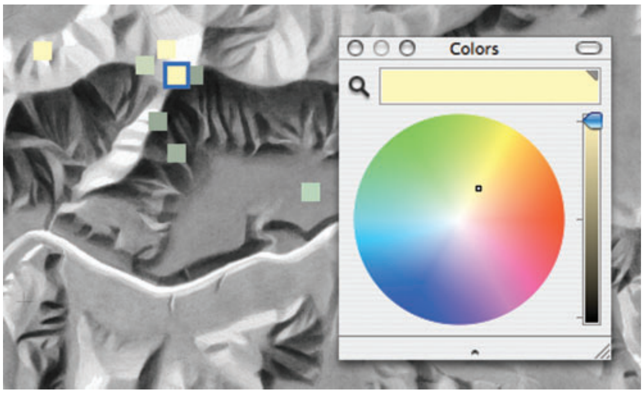

Figure 13. Screenshot of the colour picker for adjusting the colour of the selected point

(similar to Figure 10). The software then interpolates colours to complete the look-up table. However, tests show that the impact of a newly added colour point is difficult to foresee with this approach, since the colour diagram does not visually relate to the map. Fine-tuning the colour gradients is very difficult with this technique and demands a great deal of abstract thinking from the operator. The process can be simplified by customizing the colour look-up table with commercial raster graphics software (e.g., Adobe Photoshop), an approach, which is unsuitable for the creation of colour schemes from scratch. Still, minor changes to the colour nuances of an existing look-up table can be successfully performed.

The second technique uses coloured reference points that are interactively placed on the greyscale shading (Figure 12). After placing a reference point, the cartographer specifies its colour using a colour picker (Figure 13), or directly samples a colour from another image using the magnifying glass at the top left of the dialog in Figure 13. Hence, the magnifying glass allows for extracting colour from a traditional coloured relief shading, which, by the way, does not have to cover the same geographic area. For each colour reference point placed by the user, the software extracts the corresponding elevation from a georeferenced DEM, and the respective grey value from the grey shading (Figure 14). The colour points added by the user define a

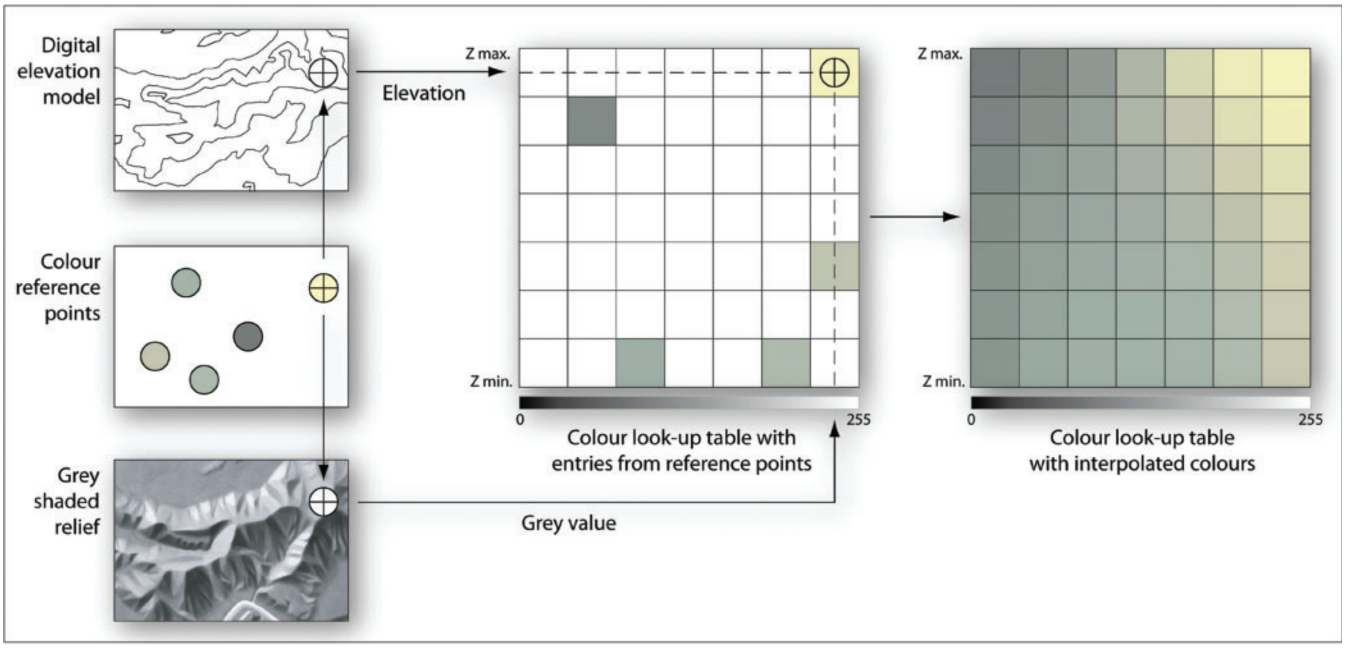

Figure 14. Initialization of the colour look-up table with colour points

Figure 15. Computer-generated colours derived from a hand-drawn grey relief shading using colour reference points and a look-up table

series of reference entries in the look-up table, which are used to interpolate colour values for the empty position in the look-up table. In case the user places two different colour points that happen to be very close to each other in the look-up table, the software resolves this ambiguity, for example by asking the user to remove one of the points. By colourizing a grey shading using a classic colour scheme developed by Imhof for school maps (Figure 15), we found that the second approach based on coloured reference points placed on the greyscale shading is more effective and user-friendly, than the first one. The two methods can also be combined, by generating a look-up table using the second approach, which can then be fine-tuned with raster graphics software, as described first.

Both methods are based on colour reference points in the look-up table that are used to interpolate colours for the rest of the table. Different methods exist for this interpolation – we chose to treat the three RGB colour components separately by interpolating three continuous surfaces (for the red, the green and the blue channel) through a set of points in a three-dimensional space. This three-dimensional space is defined by the two axes of the look-up table (the grey value along one axis and altitude along the other axis), and the colour channel along the third axis. After separately interpolating the red, green and blue channel, we assemble them to form the final image. We chose to interpolate in the RGB colour space despite the fact that the RGB space is not perceptually linear, i.e. the numerical mean of two RGB colours does not perceptually lie between the two colours. We found that interpolating in the perceptually uniform CIE-LAB or CIE-LUV colour spaces (Foley *et al.*, 1995) instead of the RGB space resulted in only slightly different colours with barely noticeable differences.

We compared four different interpolation methods using (1) triangulated irregular networks, (2) inverse distance weighting, (3) weights proportional to distance, and (4) distances weighted by a sigmoid Gaussian bell (Figure 16). In our opinion, Gaussian weighting yielded the visually best results and was simple to compute (equation 1, with $k = 0.0002$ for a table of 256 per 256 cells). A small number of colour reference points (5 to 10) proved to be sufficient with Gaussian weighting to effectively control the resulting colour scheme. We therefore used Gaussian weighting for the computer-generated example in Figures 15 and 17 (right).

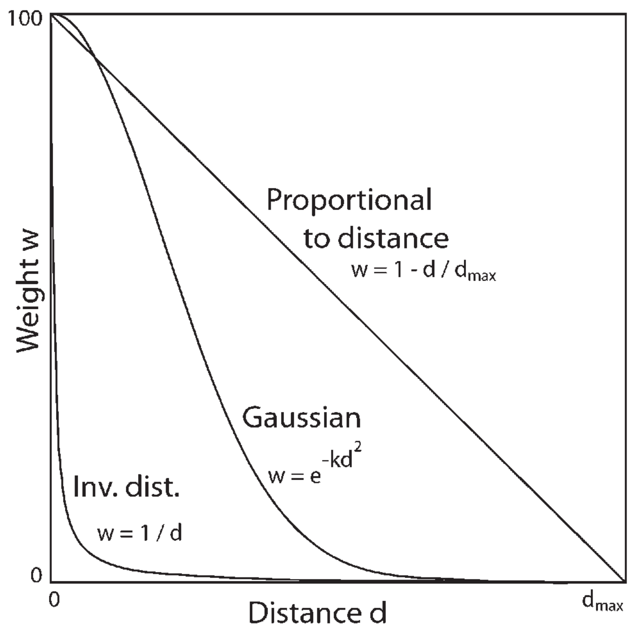

Figure 16. Weighting functions for interpolating colours in the look-up table. d is the distance in the look-up table between a colour reference point and a point to be coloured

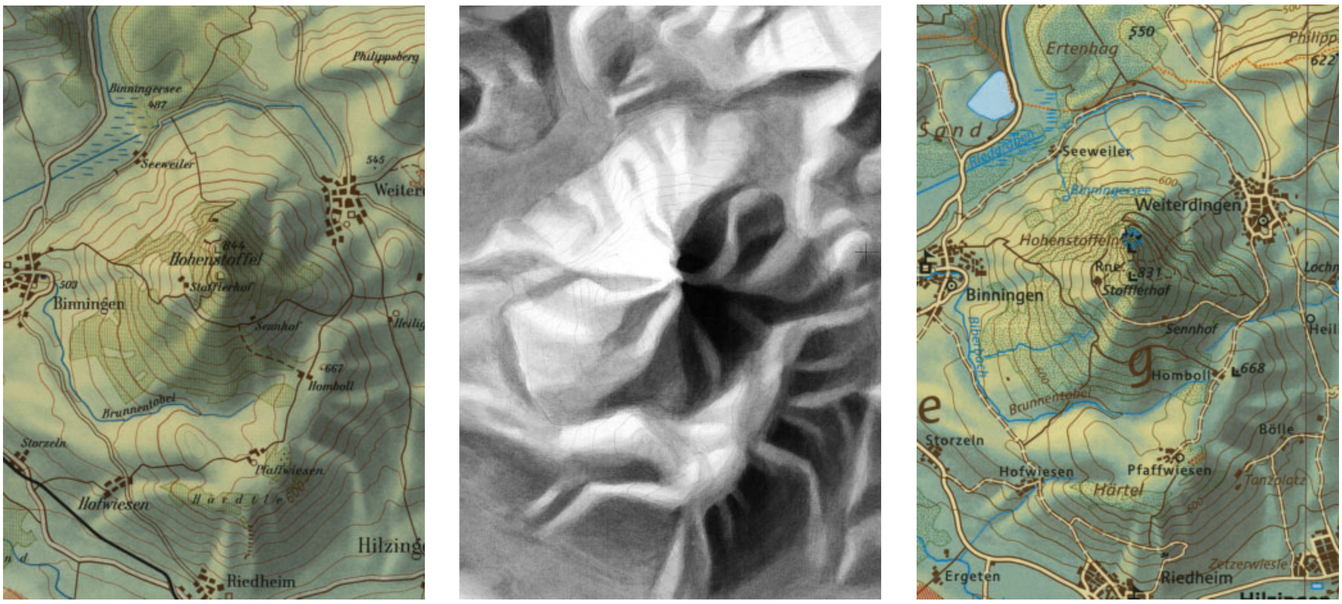

Figure 17. Schaffhausen school map of 1954 (left), the manual greyscale shading (centre), and the computer-generated edition of 2005 (right)

$$c_{xy} = \frac{\sum_i w_i \cdot c_i}{\sum_i w_i}$$

c_{xy} Colour at position x/y

c_i Colour of reference point i

w_i Gaussian weight of reference point i (see Figure 16).

Equation 1. Computation of a new colour by weighted sum of the reference colours

AN EXAMPLE: SCHAFFHAUSEN SCHOOL MAP

In order to verify the practical feasibility of the described method, a hand-drawn relief shading was colourized for a school map of the Canton of Schaffhausen. The teachers of this northern-most tip of Switzerland suggested updating the school map, which was based on a greyscale-shaded relief. They wished to revert to the colourful scheme of an earlier edition of the map, which was photomechanically produced in 1954 under supervision of Eduard Imhof (Figure 17 left). The extraordinary three-dimensional appearance of this map is due to the excellent shaded relief drawn in four parts by students of Imhof (Figure 17 centre). It was thus decided to use this original drawing and to abandon any shading produced analytically from digital elevation models. Fortunately, the shaded relief could be recovered and, after scanning, georeferencing and assembling the four parts, the tinted shading was produced in a short time using the mentioned prototype software implementing the described technique (Figure 15 and 17 right).

CONCLUSION

Since the second half of the 19th century, cartographers have developed descriptive colour schemes for printed maps. Present-day cartographers can still find valuable inspiration in the work of Leuzinger, Becker, Imhof and other pioneers, who modulated colour by elevation and by exposure to illumination. Their use of bright yellowish tints for sun-lit slopes and cooler tones for shadowed slopes successfully portrays the terrain's three-dimensionality. It is still advisable to work towards Becker's century-old goal that a brief look at a map should be enough to grasp the morphology of the terrain.

The proposed technique based on colour reference points and a look-up table has been implemented in a prototype application, which allowed us to successfully verify our approach. The examples presented prove that the quality of photomechanical colouring can be achieved using digital means. Tests with the prototype software showed that colour reference points are easy to use and simple to understand for novice users. The look-up table allows for time-effective creation of coloured relief shading, and simplifies the comparison and selection of alternative colour schemes. The method permits the colouring of traditional high-quality hand-shaded reliefs, as well as modern shadings computed from digital elevation models. A production system could offer the user a series of classic colour schemes to choose from – or offer the described concept based on colour reference points to build customized colour schemes from scratch.

Reviewing the technological development – starting with chromolithography, giving way to photomechanical reproduction, and arriving at modern digital techniques – one can note that colour schemes have always been influenced by contemporary technology. It is the authors' hope that the results presented motivate cartographers and software developers to further enhance this method, as well as other digital approaches, in order to popularize the production and use of effective and aesthetically pleasing coloured relief shading. The interested reader can find more information about various aspects of cartographic relief shading at the following web address: www.reliefshading.com (Jenny and Räber, 2004).

ACKNOWLEDGEMENTS

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MAPS

Figure 3: Becker, Fridolin, 1888. **Relief-Karte des Kantons Glarus**, 1:50,000. Winterthur: Topogr. Anst. v. Wurster, Randegger & Cie. (based on J. M. Ziegler's 'Karte von Glarus').

Figure 4: Becker, Fridolin, approx. 1900. **Rigi**, 1:50,000. Winterthur: Schlumpf.

Figure 5: Imhof, Eduard, 1954. **Kanton Schaffhausen**, 1:75,000. Bern: Kümmerly & Frey.

Figure 6: Imhof, Eduard, 1963. **Graubünden**, 1:100,000. Zürich: Art. Institut Orell Füssli.

Figure 7: Imhof, Eduard, various editions 1962–1976, copy from 1976. **Péninsule Ibérique**, 1:4 Mio. In: Atlas Scolaire Suisse pour l'Enseignement Secondaire (French edition of the Swiss Mittelschulatlas), ed. by Imhof, E., p. 62-3, Zürich: Orell Füssli Arts Graphiques.

Figure 8: [?], 1971. **Schulkarte der Schweiz** (school map of Switzerland), 1:500,000. Zürich: Art. Institut Orell Füssli. (Special print inserted in Bantel, 1973, see references).

Figure 9: Imhof, Eduard, 1982. **Relief der Schweiz – Relief de la Suisse – Rilievo della Svizzera**, 1:300,000. Wabern: Bundesamt für Landestopographie.

Figure 15 and 17 (right): Halytskyj, Joseph, and Institute of Cartography, ETH. 2005. **Kanton Schaffhausen**, 1:75,000. Schaffhausen: Erziehungsdepartement des Kantons Schaffhausen und Kantonaler Lehrmittelverlag.

Figure 17 (left): Imhof, Eduard, 1954. **Kanton Schaffhausen**, 1:75,000. Bern: Kümmerly & Frey.

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